NUCB | BUSINESS SCHOOL

Evaluation of Performance of Stock and REIT Markets in Japan

By Jiro Hodoshima

DISCUSSION PAPER NO 17302

NUCB Discussion Paper Series
March 2018

Evaluation of Performance of Stock and REIT Markets in Japan

Jiro Hodoshima*
Faculty of Economics, Nagoya University of Commerce and Business

March, 2018

Abstract

We evaluate four stock markets and one REIT market in Japan by a value measure based on utility difference pricing and the Sharpe ratio. The value measure based on utility indifference pricing responds more sensitively to the underlying risk and is more relevant for risk-averse investors than the Sharpe ratio. We reveal characteristics of the markets by the value measure and Sharpe ratio.

JEL codes; G11; G32; C13; C46; C58
Keywords; Value measure; Utility indifference pricing; Inner rate of risk aversion; Normal mixture distribution

*Corresponding author. This research was financially supported by Grant-in-Aid for Scientific Research (KAKENHI(17K03667)). Address correspondence to:Jiro Hodoshima, Faculty of Economics, Nagoya University of Commerce and Business, 4-4 Sagamine, Komenoki-cho, Nisshin-shi, Aichi 470-0193, Japan; E-mail: hodoshima@nucba.ac.jp; Tel: +81-561-73-2111 Fax: +81-561-73-1202

1 Introduction

In this paper, we evaluate performance of stock markets and a REIT market in Japan based on a value measure based on utility indifference pricing and the Sharpe ratio. Evaluation of performance of stock markets and other alternative markets is fundamental and practically important. We evaluate four stock markets and one REIT market in Japan using their value weighted indexes. Evaluation of stock markets and alternative markets is often dealt with based on performance measures such as the Sharpe ratio and information ratio, showing how big the first moment of the underlying return of a market is compared to the risk-free rate and a benchmark index adjusted for its volatility. When we try to find academic studies of stock and REIT markets in Japan by search engines, however, we can see very few of them in the literature except for reports of investment funds in the industry. This seems to be partly because it is not difficult to compute these performance measures once we have related data so that publishing with only these performance measures might have been difficult in academic journals. Furthermore, these performance measures have problems of not satisfying some desirable properties. For example, they do not satisfy monotonicity. If a value measure satisfies monotonicity, it holds that a value measure of $\boldsymbol{X}$ is greater than and equal to that of $\boldsymbol{Y}$ when $\boldsymbol{X} \geq \boldsymbol{Y}$ almost surely where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are random variables of two financial asset returns (see, e.g., Zhitlukhin (2014)). In this paper, we provide evaluation of performance of stock markets and a REIT market using a value measure based on utility indifference pricing which satisfies desirable properties including monotonicity.

The utility indifference price of a random variable $\boldsymbol{X}$ denoting an asset return is defined to be the solution ν of the equation $E[u(-\nu + \boldsymbol{X})] = 0$ where $u(\cdot)$ denotes a utility function and E denotes expectation. When $\boldsymbol{X}$ increases almost surely, $\boldsymbol{X}$ becomes more desirable and ν also increases, which has been proven by Miyahara (2010). Hence ν, the utility indifference price, satisfies monotonicity and is a value measure of $\boldsymbol{X}$. There are not many studies based on the utility indifference price to evaluate performance of stocks. Miyahara (2010) showed the utility indifference price satisfies several desirable properties including monotonicity and concavity a suitable evaluation function ought to satisfy. The several desirable properties a suitable evaluation function ought to satisfy are explained in the next section. Concavity is a risk-averse property which implies the law of diminishing marginal utility. Cherny and Madan (2008) characterized performance measures satisfying a set of desirable properties, which is somewhat different from the properties we use in this paper. Furthermore it is shown (given as Proposition 2 in the next section) that the exponential utility function $u(x) = \frac{1}{\alpha}(1 - e^{-\alpha x})$ and its associated utility indifference price is the only utility function and the only utility indifference price among $\boldsymbol{C}^2$-class[1] of utility functions under some condition where α is the degree of risk aversion. Therefore, the utility indifference price with the exponential utility function is the only possible candidate for the suitable evaluation function and Miyahara (2010) called it a "risk-sensitive value measure" (RSVM) because it gives values sensitively according to the underlying characteristic of $\boldsymbol{X}$, i.e., it responds a positive value if $\boldsymbol{X}$ is profitable or attractive and a negative value if $\boldsymbol{X}$ is undesirable. The RSVM of a random variable of a stock return $\boldsymbol{X}$ can be easily seen to be given by

$$-\frac{1}{\alpha} \ln E[e^{-\alpha \boldsymbol{X}}].$$

Therefore, we can obtain the RSVM explicitly, which makes its computation easy. Typically, if the degree of risk aversion α increases, i.e., if an investor with the exponential utility function becomes more risk-averse, the RSVM of $\boldsymbol{X}$ decreases.

We employ a value measure based on the RSVM, which is named internal rate of risk aversion (IRRA) by Miyahara (2014), in order to evaluate performance of stock markets and a REIT market. The IRRA of $\boldsymbol{X}$ is the degree of risk aversion that makes zero the RSVM, i.e., the IRRA of $\boldsymbol{X}$ is given by the solution α_0 of the equation

$$-\frac{1}{\alpha_0} \ln E[e^{-\alpha_0 \boldsymbol{X}}] = 0.$$

The larger the value of the RSVM of $\boldsymbol{X}$ is, the more desirable $\boldsymbol{X}$ is. As the IRRA increases, the degree of risk aversion increases that makes zero the RSVM. When the IRRA is large for an asset return $\boldsymbol{X}$, then $\boldsymbol{X}$ becomes attractive for many investors since the range of degree of risk aversion under which the RSVM is positive becomes large and hence it becomes desirable for many risk-averse investors. Therefore, the IRRA is also a

[1] $\boldsymbol{C}^2$-class is the class of functions that possess continuous second derivatives.

value measure. We remark $\boldsymbol{X}$ is desirable (undesirable) when the RSVM of $\boldsymbol{X}$ is positive (negative) by the definition and property of the RSVM described in the next section. The IRRA is derived from the RSVM which is the only candidate for the suitable value measure. Therefore, we expect evaluation based on the IRRA to be reasonable. The RSVM depends on the degree of risk aversion and hence we must choose the degree of risk aversion to determine the RSVM. On the other hand, the IRRA itself is the degree of risk aversion so that we do no need to specify the degree of risk aversion to determine the IRRA. We use the IRRA to evaluate performance of stock and REIT markets. Alternatively, one can fix the degree of risk aversion to determine the RSVM to be used to evaluate performance of stock and alternative markets.

We compare evaluation of stock markets and a REIT market by the IRRA and Sharpe ratio in this paper. We study four stock markets and one REIT market in Japan. By comparing these markets based on the IRRA and Sharpe ratio, we show which market is desirable with respect to the suitable risk-averse value measure and the *de facto* industry standard performance measure of the Sharpe ratio among the five markets. Evaluation of the markets based on the IRRA provides an evaluation of a risk-averse investor with the exponential utility function, which seems to be relevant for risk-averse investors. Comparison of these markets by the two measures reveals the different nature of the two measures and shows relevance of evaluation based on the IRRA for risk-averse investors compared to that based on the Sharpe ratio. The IRRA has been applied to evaluate performance of individual Hong Kong REITs (see Ban et al. (2016)) and individual stocks in the U.S. and Japan (see Hodoshima et al. (2017a, 2017b)).

In order to calculate the RSVM and IRRA for stock and REIT markets, we assume the distribution of the stock and REIT market returns to follow the class of discrete normal mixture distributions. The class of discrete normal mixture distributions is well known to be flexible to describe characteristics of not only symmetric distributions but also leptokurtic and skewed distributions often observed in financial data. It has been studied for several decades in the statistics and econometrics literature (cf, e.g., Everitt and Hand (1981), Titterington et al. (1985), and McLachlan and Peel (2000)). There are also many financial applications of discrete normal mixture distributions (cf., e.g., Kon (1984), Ritchey (1990), Chin et al. (1999), Brigo and Mercurio (2001), Alexander (2004), and Hodoshima et al. (2018)). Therefore, it is not unnatural to assume the class of discrete normal mixture distributions to describe stock and REIT market return data in our empirical study. By assuming the class of discrete normal mixture distributions, we can obtain, without much difficulty, the RSVM and IRRA once we estimate parameters of the best fit normal mixture distribution as shown in Section 3. We remark a method of computing the IRRA by first estimating the RSVM by a method of moments estimator directly from data does not work when the RSVM thus estimated is always positive and/or negative for any value of the degree of risk aversion (cf. Ban et al. (2016)). Failure of obtaining the IRRA by this direct method can often happen depending on underlying data. Our method of obtaining the IRRA by assuming the class of discrete normal mixture distributions makes operational derivation of the IRRA and hence it is a practical method of computing the IRRA.

The paper is organized as follows. Section 2 introduces definitions and properties of the RSVM and IRRA proposed by Miyahara (2010, 2014, 2017). Section 3 presents a definition and properties of the class of discrete normal mixture distributions and shows how to obtain the RSVM, IRRA and Sharpe ratio for the random variable of an asset return under the class of discrete normal mixture distributions. Section 4 shows estimation results of Japanese stock and REIT markets and provides comparison results based on the IRRA and Sharpe ratio for stock and REIT markets in Japan. Section 5 provides concluding comments.

2 The RSVM and IRRA

In this section we provide definitions and properties of the RSVM and IRRA. First we provide the definition of a concave monetary value measure. The concept of a concave monetary value measure (or concave monetary utility function) has been introduced in Cheridito et al. (2006).

Definition 1 (Concave Monetary Value Measure) *A functional $v(\boldsymbol{X})$ defined on a space $\boldsymbol{L}$ of random variables denoted as $\boldsymbol{X}$ is called a concave monetary value measure if it satisfies the following conditions,*

1. *(Normalization)* $v(0) = 0$,

2. *(Monotonicity) If* $\boldsymbol{X} \leq \boldsymbol{Y}$*, then* $v(\boldsymbol{X}) \leq v(\boldsymbol{Y})$*, where* $\boldsymbol{X}$ *and* $\boldsymbol{Y}$ *are both random variables,*

3. *(Translation invariance, or Monetary property)* $v(\boldsymbol{X} + m) = v(\boldsymbol{X}) + m$*, where* m *is non-random,*

4. *(Concavity)* $v(\lambda \boldsymbol{X} + (1-\lambda)\boldsymbol{Y}) \geq \lambda v(\boldsymbol{X}) + (1-\lambda) v(\boldsymbol{Y})$ *for* $0 \leq \lambda \leq 1$,

5. *(Law invariance)* $v(\boldsymbol{X}) = v(\boldsymbol{Y})$ *whenever law(*$\boldsymbol{X}$*) = law(*$\boldsymbol{Y}$*).*

In Definition 1, law($\boldsymbol{X}$) denotes law of $\boldsymbol{X}$, i.e., distribution of $\boldsymbol{X}$. Normalization and translation invariance are not included in the axioms for desirable performance measures by Cherny and Madan (2008). Normalization is convenient to categorize a random variable with a positive (negative) value measure as desirable (undesirable). Translation invariance is a natural requirement for a value measure. Instead of concavity given in Definition 1, Cherny and Madan (2008) included quasi-concavity in their axioms for desirable performance measures.

Remark *Set* $\lambda = 1/2$ *and* $\boldsymbol{Y} = -\boldsymbol{X}$ *in the concavity condition. Then we have*

$$v(\frac{1}{2}\boldsymbol{X} + \frac{1}{2}(-\boldsymbol{X})) \geq \frac{1}{2}v(\boldsymbol{X}) + \frac{1}{2}v(-\boldsymbol{X}).$$

The left hand side of the above equation is equal to 0, which implies

$$v(\boldsymbol{X}) \leq -v(-\boldsymbol{X}).$$

When $v(\boldsymbol{X}) > 0$*, the above inequality implies* $v(-\boldsymbol{X}) < 0$ *and* $|v(-\boldsymbol{X})| \geq v(\boldsymbol{X})$*. In other words, the investor who obeys the concave monetary value measure is more sensitive to the loss of* $\boldsymbol{X}$ *being negative than the gain of* $\boldsymbol{X}$ *being positive.*

The property described in Remark is equivalent to the law of diminishing marginal utility (cf., e.g., page 356 of Layard and Walters (1978)). Then, large negative observations of $\boldsymbol{X}$ can make its RSVM disproportionally small.

Next we define a utility function.

Definition 2 (Utility Function) *A real valued function $u(x)$ defined on $\mathcal{R}^1$ is called a utility function if it satisfies the following conditions,*

1. *$u(x)$ is continuous and non-decreasing,*

2. *$u(x)$ is concave,*

3. *$u(0) = 0$.*

We then define the utility indifference price in a utility function $u(x)$ as follows.

Definition 3 (Utility Indifference Price) *The utility indifference price of $\boldsymbol{X}$, denoted as $UIP(\boldsymbol{X})$, is the solution of the following equation*

$$E[u(\boldsymbol{X} - UIP(\boldsymbol{X}))] = u(0) = 0.$$

The utility indifference price is an evaluation of $\boldsymbol{X}$ by an investor who has a utility function $u(\cdot)$ given in the definition. Thus the utility indifference price is an evaluation of $\boldsymbol{X}$ for the investor who has a utility function $u(\cdot)$.

Miyahara (2010) proved the utility indifference price is a concave monetary value measure, which we give as the following proposition.

Proposition 1 *The utility indifference price $UIP(\boldsymbol{X})$ is a concave monetary value measure.*

Therefore, the utility indifference price is desirable since it satisfies all the conditions of the concave monetary value measure.

When the utility function is an exponential type utility function given by $u(x) = \frac{1}{\alpha}(1-e^{-\alpha x})$ where α denotes the degree of risk aversion with $\alpha > 0$, the utility indifference price $UIP(\boldsymbol{X})$ can be easily seen to be given by

$$UIP(\boldsymbol{X}) = -\frac{1}{\alpha}\ln E[e^{-\alpha \boldsymbol{X}}]. \tag{1}$$

Miyahara (2010) called the above utility indifference price the RSVM because it provides a value according to the underlying distribution of $\boldsymbol{X}$ and hence it is sensitive to the underlying risk of $\boldsymbol{X}$ as compared to traditional value measures such as the mean-variance (MV) approach.

It can be seen that the RSVM of $\boldsymbol{X}$, i.e., $-\frac{1}{\alpha}\ln E[e^{-\alpha \boldsymbol{X}}]$, is equal to the MV approach, defined by

$$MV(\alpha) = E[\boldsymbol{X}] - \frac{\alpha}{2}V[\boldsymbol{X}] \tag{2}$$

when the random variable $\boldsymbol{X}$ is normally distributed where $V[\boldsymbol{X}]$ denotes variance of $\boldsymbol{X}$. Thus the RSVM is in a sense a generalization of the traditional value measure of the MV approach. It gives evaluations, compared to the MV approach, sensitively when the underlying distribution is asymmetric as well as heavy-tailed (see Hodoshima et al. (2018)). This is a main advantage of our approach compared to the MV approach. It can be also shown that both the RSVM of $\boldsymbol{X}$ and $MV(\alpha)$ approach to $E[\boldsymbol{X}]$ as α goes to zero. Most importantly, the MV approach does not satisfy the concave monetary value measure and hence the RSVM is more desirable than the MV approach.

We now show the exponential utility function $u(x) = \frac{1}{\alpha}(1 - e^{-\alpha x})$ is a special utility function. First we introduce an additivity condition for independent random variables.

Definition 4 (Independence-additivity) *If an evaluation function $v(x)$ satisfies the following condition*

(Additivity) If $\boldsymbol{X}$ and $\boldsymbol{Y}$ are independent, then $v(\boldsymbol{X}+\boldsymbol{Y}) = v(\boldsymbol{X}) + v(\boldsymbol{Y})$,

then $v(x)$ is said to satisfy the independence-additivity condition.

It can be easily seen that the RSVM satisfies the independence-additivity condition.

Then we have the following result about the exponential utility function.

Proposition 2 *Assume that the utility function $u(x)$ is of $\boldsymbol{C}^2$-class, increasing, concave, normalized as $u(0) = 0, u'(0) = 1$, and $u''(0) = \alpha$ where $\boldsymbol{C}^2$-class is the class of functions that possess continuous second derivatives. And assume that the utility indifference price under a utility function $u(x)$ satisfies the independence-additivity condition. Then $u(x)$ is of the following form*

$$u(x) = \frac{1}{\alpha}(1 - e^{-\alpha x}).$$

See Theorem 3.2.8 of Rolski et al. (1999) for its proof. This implies the exponential utility function and its utility indifference price, i.e., the RSVM, is the only utility function and the only UIP among $\boldsymbol{C}^2$-class of utility functions under the condition of

Proposition 2. Since the UIP is a concave monetary value measure as shown by Proposition 1, then it can be said that the RSVM, the UIP with the exponential utility function, is the only possible candidate for the suitable value measure.

We now present definition of the IRRA proposed by Miyahara (2014).

Definition 5 (Internal Rate of Risk Aversion (IRRA)) *Let $\boldsymbol{X}$ be a random variable denoting an asset return. Internal rate of risk aversion (IRRA) of $\boldsymbol{X}$ is defined by the solution α_0 of the equation $-\frac{1}{\alpha_0}\ln E[e^{-\alpha_0 \boldsymbol{X}}] = 0$, i.e., the degree of risk aversion that makes zero the utility indifference price with the exponential utility function $u(x) = \frac{1}{\alpha}(1 - e^{-\alpha x})$.*

A sufficient condition of existence and uniqueness of the IRRA was provided by Miyahara (cf., Proposition 9 of Miyahara (2014)). We present it as follows.

Proposition 3 *Assume the moment-generating function (MGF) of $\boldsymbol{X}$ exists and if the following conditions are satisfied, i.e.,*

$$E[\boldsymbol{X}] > 0 \quad \text{and} \quad P(\boldsymbol{X} < 0) > 0,$$

then the IRRA $\alpha_0(> 0)$ exists and is unique.

Proof of Proposition 3 was given in Miyahara (2017) and is provided at Internet Appendix (not for publication). Since the IRRA α_0 is positive under the conditions of Proposition 3, the IRRA of $\boldsymbol{X}$ is equivalent to the solution of the equation $E[e^{-\alpha_0 \boldsymbol{X}}] = 1$. Suppose performance of $\boldsymbol{X}$ is good so that a positive part of $\boldsymbol{X}$ weighs more than a negative part of $\boldsymbol{X}$. A positive value of $\boldsymbol{X}$ makes $e^{-\alpha_0 \boldsymbol{X}}$ less than one since α_0 is positive. Then, if a positive part of $\boldsymbol{X}$ weighs more than a negative part of $\boldsymbol{X}$, $e^{-\alpha_0 \boldsymbol{X}}$ being less than one weighs more than $e^{-\alpha_0 \boldsymbol{X}}$ being larger than one and hence α_0 needs to be larger in order for α_0 to satisfy the equation $E[e^{-\alpha_0 \boldsymbol{X}}] = 1$. Therefore, if performance of an underlying financial target is good (bad), then its associated IRRA is high (low). Thus the IRRA becomes a value measure of performance of the underlying financial target. We use it to value performance of stock and REIT markets in Japan in Section 4. Since the IRRA is based on the RSVM, which is the only candidate for the suitable value measure as seen above, we expect it will provide sensible evaluation of performance of stock and alternative markets.

3 Discrete Normal Mixture Distributions

We assume the distribution of stock and REIT market index returns to follow the class of discrete normal mixture distributions in order to obtain the IRRA of performance of these market indexes. Below we give the definition of discrete normal mixture distributions.

The probability density function (pdf) of a mixture of K normal distributions is given by

$$f(x) = \sum_{i=1}^{K} \pi_i \phi(x; \mu_i, \sigma_i) \tag{3}$$

where, for $i = 1, \cdots, K$,

$$\begin{aligned} \phi(x; \mu_i, \sigma_i) &= \frac{1}{\sqrt{2\pi}\sigma_i} \exp(-\frac{(x-\mu_i)^2}{2\sigma_i^2}) \\ \sum_{i=1}^{K} \pi_i &= 1 \quad \text{and} \quad 0 \leq \pi_i \leq 1. \end{aligned}$$

If $\boldsymbol{X}$ is a mixture of K normal distributions with pdf (3), then its mean, variance, skewness, and kurtosis are given respectively by

$$\begin{aligned} \mu &= \sum_{i=1}^{K} \pi_i \mu_i \\ \sigma^2 &= \sum_{i=1}^{K} \pi_i (\sigma_i^2 + \mu_i^2) - \mu^2 \\ \tau &= \frac{1}{\sigma^3} \sum_{i=1}^{K} \pi_i (\mu_i - \mu) \left[3\sigma_i^2 + (\mu_i - \mu)^2\right] \\ \kappa &= \frac{1}{\sigma^4} \sum_{i=1}^{K} \pi_i \left[3\sigma_i^4 + 6(\mu_i - \mu)^2 \sigma_i^2 + (\mu_i - \mu)^4\right]. \end{aligned} \tag{4}$$

As is well known, the MGF $E[e^{t\boldsymbol{Y}}]$ of a random variable $\boldsymbol{Y}$ whose pdf is the normal density $\phi(y; \mu, \sigma)$ is given by

$$\exp(\mu t + \sigma^2 t^2/2). \tag{5}$$

Observe $E[e^{-\alpha\boldsymbol{X}}]$ in the RSVM of $\boldsymbol{X}$ given by the formula (1) is, besides the minus sign, nothing but the MGF of $\boldsymbol{X}$. Therefore, if $\boldsymbol{X}$ is a mixture of K normal distributions with pdf (3), its RSVM is given by

$$-\frac{1}{\alpha}\ln[\sum_{i=1}^{K}\pi_i\exp(-\mu_i\alpha+\sigma_i^2\alpha^2/2)]. \tag{6}$$

Thus, when the underlying distribution of $\boldsymbol{X}$ is given by a mixture of K normal distributions, the RSVM can be obtained analytically by (6). Once the RSVM is formulated by (6), the IRRA can be obtained by solving for α_0 of the following equation

$$-\frac{1}{\alpha_0}\ln[\sum_{i=1}^{K}\pi_i\exp(-\mu_i\alpha_0+\sigma_i^2\alpha_0^2/2)]=0. \tag{7}$$

Thus, the IRRA can be derived when the underlying distribution is in the class of discrete normal mixture distributions provided that the IRRA exists.

The formulas (6) and (7) imply the following properties of the RSVM and IRRA of $\boldsymbol{X}$ if other conditions are equal when the underlying distribution of $\boldsymbol{X}$ is given by a mixture of K normal distributions with pdf (3).

- Positive (negative) mean in a component makes the RSVM and IRRA large (small) .
- Large (small) variance in a component makes the RSVM and IRRA small (large).

In particular, the effect of variance in a component on the RSVM and IRRA is proportional to the square of that of mean in the same component, which can be seen in the formulas (6) and (7). Therefore, large variance in a component makes the RSVM and IRRA disproportionally small compared to negative mean in the same component if other conditions are equal.

Similarly the Sharpe ratio can be derived using mean and variance formulas given in equation (4) when the distribution is to follow the class of discrete normal mixture distributions.

4 Performance of Japanese stock and REIT markets by the IRRA and Sharpe ratio

In this section, we investigate performance of stock and REIT markets in Japan by the two value measures of the IRRA and Sharpe ratio. As stock markets in Japan, we study the first section and second section of the Tokyo Stock Exchange (TSE), the TSE Mothers Securities Exchange, and the TSE JASDAQ Securities Exchange. They are major stock markets in Japan. We study daily time series indexes of market capitalization of these four stock markets as compared to market capitalization at certain time points. In other words, we use TOPIX (Tokyo Stock Price Index), TSE Second Section Index, TSE Mothers Index, and TSE JASDAQ Index, which are indexes of the first section of the TSE, the second section of the TSE, TSE Mothers Exchange, and TSE JASDAQ Exchange respectively. The stock market with the strictest listing requirements is the first section of the TSE, followed in order by the second section of the TSE, the TSE JASDAQ Exchange, and the TSE Mothers Exchange. As listing requirements are stricter, companies listed in the market with the listing requirements become generally larger in size. We use daily returns from closing prices of these indexes. Similarly we study a daily time series index of market capitalization of REIT funds listed in the TSE as compared to market capitalization at March 31, 2003. It is named the REIT Index. We also use daily returns from daily closing prices of the REIT index. We aim to obtain the two value measures of the IRRA and Sharpe ratio of the return data of the five indexes of stock and REIT markets in Japan. We use return data from January 4, 2008 till April 28, 2017, the period including the global financial crisis up to a recent time. We can compare evaluation of the Japanese stock and REIT markets in this paper with that of individual stocks in the U.S. in Hodoshima et al. (2017a) and Japan in Hodoshima et al. (2017b), which cover the same sample period based on the same two measures of the IRRA and Sharpe ratio. In order to calculate the Sharpe ratio, we use the uncollateralized overnight call rate, available from Bank of Japan, as the daily risk-free rate. The stock and REIT index data are downloaded from a stock market data site of http://k-db.com[2].

Summary statistics of the five stock and REIT indexes are given at Table 1. The five indexes have small positive means with the range from 0.009 in REIT to 0.038 in Mothers. Standard deviation ranges from 0.932 in the 2nd section of the TSE to 2.250 in Mothers. They are all negatively skewed and heavy tailed distributions. The Tokyo

[2]We are grateful to this site for making data available to everyone.

Stock Second Section Index and JASDAQ Index appear to be similar with respect to mean and standard deviation although kurtosis in the JASDAQ Index is almost half of that in the Tokyo Stock Second Section Index. The Mothers Index has the largest mean and the largest standard deviation among the five markets so the TSE Mothers Exchange is a market with high return and high risk. The REIT has the smallest mean and its standard deviation is similar to that in the TOPIX.

Figures of the five market index prices are given at Figures 1-5. Index prices begin on December 28, 2007. Figure 1-5 correspond respectively to figure of the TOPIX, TSE Second Section Index, TSE Mothers Index, and TSE JASDAQ Index, and TSE REIT Index. The TOPIX pushed its price down more than 40 % during the global financial crisis compared to early 2008 but later increased its price up more than twice as much as the bottom price. The TSE Second Section pushed its price down about 30 % during the global financial crisis but later increased it about three times as much as the bottom price during the global financial crisis. It appeared more vigorous than the TOPIX. The Mothers Index pushed its price down more than 50 % during the global financial crisis and later attained the highest price four times as much as the bottom price during the global financial crisis. However, the Mothers Index appears to be unstable in the recovering period. The JASDAQ Index appears to have experienced a course similar to the TSE Second Section. The REIT Index appears to perform worst among the five indexes; it had, during the global financial crisis, its price lower than half of a peak price in early 2008 and later recovered with its price not much different from the peak price of early 2008.

Then we obtain the two measures of the IRRA and Sharpe ratio for the stock and REIT market indexes. First we fit the class of discrete normal mixture distributions to the five stock and REIT market index data. In order to fit a discrete normal mixture distribution to the data, we use an expectation-maximum (EM) algorithm (see, e.g., Hastie et. al. (2003)) to obtain parameter estimates of a discrete normal mixture distribution. To obtain parameter estimates of a discrete normal mixture distribution, we need to determine the number of components of discrete normal mixture distributions. We use Bayesian Information Criterion (BIC) to determine the number of components of discrete normal mixture distributions. We give the value of BIC at Table 2 for two, three, and four components mixture distributions in the five stock and REIT market index data. We choose to use the number of components whose BIC is minimum. We cannot obtain BIC for a four components normal mixture distribution in the case of the TOPIX and REIT Index because the EM algorithm does converge[3] for a four components normal mixture distribution in the TOPIX and REIT Index. A two components normal mixture distribution is chosen in the case of the TOPIX and Mothers Index and a three components normal mixture distribution is chosen in the rest of the indexes. Parameter estimates of the best fit normal mixture distribution are given at Table 3. Although the best fit normal mixture distribution is either a two or a three components normal mixture distribution, it is generally composed of one component which indicates a negative shock state with negative mean, large variance, and small probability as well as other components which show more stable days with positive or larger mean, smaller variance, and larger probability.

Once we obtain parameter estimates of the best fit normal mixture distribution, we can obtain its RSVM and IRRA by the formulas (6) and (7) in the previous section. We present the IRRA at Table 4 for the five stock and REIT market indexes. We also present the Sharpe ratio for the five indexes based on mean and standard deviation given in summary statistics and also mean of the risk-free rate. The Sharpe ratio is given at Table 5 for the five indexes. We remark the Sharpe ratio can be also obtained from mean and standard deviation formulas of the best fit normal mixture distribution given in equation (4) in the previous section. However, the results virtually do not change with respect to values up to the third decimal point shown in Table 5. This indicates the best fit normal mixture distribution can capture well characteristics of the five indexes so that it can reproduce virtually the same values of the Sharpe ratio as those derived from mean and standard deviation of the data for the five indexes.

We now interpret outcomes of the ranking of the IRRA and Sharpe ratio given at Table 4 and 5. We first evaluate the ranking of the Sharpe ratio. The Sharpe ratio is 0.005, 0.008, and 0.016 respectively for the REIT, Mothers, and TOPIX. On the other

[3]The EM algorithm does not converge within a 10000 iteration limit in our experiment.

hand, in the preceding study of the Dow Jones Industrial Average Index (DOW) and Nasdaq Composite Index (NASDAQ) in the same sample period by Hodoshima et al. (2017a), the Sharpe ratio is 0.021 and 0.031 respectively for the DOW and NASDAQ although the DOW is a different type of index. Therefore, these three markets did not perform well in terms of the Sharpe ratio. However, the Sharpe ratio is 0.029 and 0.031 respectively for the JASDAQ and TSE Second Section. Thus performance of these two markets is similar to that of the DOW and NASDAQ. In a preceding study of Hodoshima et al. (2017b), performance of selective Japanese individual stocks are studied based on the IRRA and Sharpe ratio. The individual stocks studied there are all listed in the first section of the TSE and they are all big companies in size. Their Sharpe ratios are mostly small with evaluations less than 0.02. Softbank, Keyence, and Sysmex studied in Hodoshima et al. (2017b) are three exceptions with high Sharpe ratios, respectively 0.034, 0.039, and 0.046, which are higher than the JASDAQ and Tokyo Stock Exchange Second Section.

Next we evaluate the ranking of the IRRA. The IRRA is 0.007, 0.011, and 0.015 respectively for the REIT, Topix, and Mothers Index. These IRRAs are not much different from the corresponding Sharpe ratios. This implies evaluation by the IRRA, i.e., an evaluation by a risk-averse investor with the exponential utility function, is similar to that by the direct performance measure of the Sharpe ratio. On the other hand, the IRRA is 0.037 and 0.046 for the DOW and NASDAQ studied in Hodoshima et al. (2017a). Therefore, the three indexes of the REIT, TOPIX, and Mothers do not perform well compared to the DOW and NASDAQ in the U.S. with respect to the IRRA. However, the IRRA is 0.054 and 0.065 for the JASDAQ and TSE Second Section respectively. The IRRA is larger than the Sharpe ratio for these two markets, which implies the two markets of the JASDAQ and TSE Second Section perform better based on an evaluation by a risk-averse investor than a direct evaluation by the Sharpe ratio. It also implies the Japanese two markets of the JASDAQ and TSE Second Section perform better than the U.S. two markets of the DOW and NASDAQ based on an evaluation by a risk-averse investor with the exponential utility function. The TSE second section and and JASDAQ Securities Exchange are markets where smaller companies are listed compared to the first section of the TSE where bigger and well-known companies are listed. The results of these two markets with respect to the IRRA are provided for the first time and shed a new light from a risk-averse evaluation with the exponential utility function. Large Japanese individual stocks studied in Hodoshima et al. (2017b) mostly have the IRRAs less than 0.03 with exception of Keyence 0.038 and Sysmex 0.043. Thus the IRRAs of the two best performing companies studied in Hodoshima et al. (2017b), both listed in the first section of the TSE, are less than those of the JASDAQ and TSE Second Section. Therefore both the JASDAQ and TSE Second Section also perform better than the best performing individual companies studied in Hodoshima et al. (2017b).

We consider the following possible reason of the difference of these five stock and REIT markets in evaluation of the IRRA. In other words, we consider there are two reasons; (1) influence of foreign investors and (2) difference of listing requirements. The first reason is influence of foreign investors on different markets who tend to earn short-term profits compared to domestic investors. Frequent trades by foreign investors make markets volatile, which makes the IRRA small as shown in the previous section. In the largest market of the first section of the TSE, about 70 % of transactions are made by foreign investors (see JAPAN EXCHANGE GROUP Data & Statistics, Trading by Type of Investors http://www.jpx.co.jp/english/markets/statistics-equities/investor-type/index.html) so that the first section of the TSE is often controlled by foreign investors. On the other hand, influence of foreign investors is less in other markets. At most 30 % of trades are made by foreign investors in other stock markets with the exception of the second section of the TSE in 2017. In the case of the TSE Mothers Exchange, listing requirements are loose and sales as well as profits are more uncertain compared to other stock markets. That seems to be partly the reason why Mothers is not performing well. In the case of the REIT market, about 50 % of trades are made by foreign investors, which makes the REIT market unstable. On the other hand, the JASDAQ and TSE second section are markets where domestic individual investors are dominant players, which makes these markets more stable compared to other markets. Then their IRRAs will increase as seen in the previous section.

In addition to the results obtained above, we introduce an additional evidence on the JASDAQ Index. The TSE JASDAQ market is composed of companies listed in the JASDAQ standard section and JASDAQ growth section. Listing requirements are stricter in the JASDAQ standard section than in the JASDAQ growth section. About 95 % of companies in the TSE JASDAQ Exchange are listed in the JASDAQ standard section with the remainder in the JASDAQ growth section. Indexes for these two submarkets have been created in October 2010. They are the JASDAQ Index (Standard) (JASDAQ standard) and JASDAQ Index (Growth) (JASDAQ growth) which are time series indexes of market capitalization of these two stock submarkets as compared to market capitalization at time point in October 2010. We use daily returns from closing prices of these indexes from October 15, 2010 till April 28, 2017. To compare with these two indexes, we also examine performance of the NASDAQ in the U.S. during the same sample period.

Summary statistics are given at Table 6 for the JASDAQ standard, JASDAQ growth, and NASDAQ. The three indexes have positive means with the range from 0.060 in NASDAQ to 0.093 in JASDAQ growth. Standard deviation ranges from 1.045 in NASDAQ to 2.651 in the JASDAQ growth. The JASDAQ growth is a market with high risk and high return compared to other two markets. In fact, the JASDAQ growth section is a market with the weakest listing requirements, weaker than the TSE Mothers Exchange. Listing requirements are stricter in the JASDAQ standard than in the Mothers, which is stricter than in the JASDAQ growth. They are all negatively skewed and heavy tailed distributions. The JASDAQ standard and NASDAQ appear to be similar with respect to mean and standard deviation.

Figures of the three market index prices are given at Figures 6-8. Figure 6-8 correspond respectively to figure of the JASDAQ standard, JASDAQ growth, and NASDAQ. The JASDAQ standard increased its price in April 2017 nearly three times as much as the beginning price in October 2010. The JASDAQ growth increased its price in 2013 up to seven times as much as the beginning price in October 2010 but ended up in April 2017 with the price twice as much as the beginning price. The NASDAQ increased its price in April 2017 up to 2.4 times as much as the beginning price.

We first fit the class of discrete normal mixture distributions to the daily return data of the JASDAQ standard, JASDAQ growth, and NASDAQ. We provide the value of BIC at Table 7 for two, three, and four components mixture distributions in the JASDAQ standard, JASDAQ growth, and NASDAQ. The best fit normal mixture distribution is a two-components normal mixture distribution for the three indexes. Parameter estimates of the best fit normal mixture distribution are given at Table 8. The JASDAQ standard and NASDAQ have a shock state with negative mean, larger variance, and smaller probability and a stable state with positive mean and smaller variance. Hence they are, in terms of the characteristic of the best fit normal mixture distribution, similar to the five stock and REIT markets described above. On the other hand, the JASDAQ growth has a positive shock with larger positive mean, larger variance, and smaller probability.

Table 9 and 10 provide respectively the IRRA and Sharpe ratio for the three indexes. The JASDAQ growth has the smallest IRRA and Sharpe ratio. Its IRRA is less than its Sharpe ratio, indicating its risk-averse evaluation is less than the de facto industry standard performance measure. This implies its performance is less desirable for risk-averse investors than the evaluation the Sharpe ratio provides. On the other hand, the IRRA is higher than the Sharpe ratio for the JASDAQ standard and NASDAQ, suggesting their performance is more desirable for risk-averse investors than that based on the Sharpe ratio. Both the IRRA and Sharpe ratio are much higher in the JASDAQ standard and NASDAQ than in the JASDAQ growth. As both the IRRA and Sharpe ratio for the JASDAQ standard is higher than those for the NASDAQ, performance of the JASDAQ standard is better for risk-averse investors as well as in terms of the direct performance measure of the Sharpe ratio than the NASDAQ gathering the most attention in the world. Since Japanese stock markets are known to have recently performed rather poorly compared to the rest of the world, this is a surprising fact. Our examination of the JASDAQ standard, JASDAQ growth, and NASDAQ is in a sense a subsample test of our findings of the five stock and REIT markets for the first sample period from January 4, 2008 till April 28, 2017. Good performance of the JASDAQ standard based on the risk-averse value measure is confirmed again in comparison to the world-leading NASDAQ.

5 Concluding comments

We have evaluated performance of the stock and REIT markets in Japan based on the IRRA and Sharpe ratio. The IRRA is a performance measure based on the RSVM which is the only candidate for the suitable value measure. It provides an evaluation based on a risk-averse investor's assessment which can be more valuable for risk-averse investors than the de facto industry standard of the Sharpe ratio. We have obtained the IRRA by assuming the underlying return distribution to follow the class of discrete normal mixture distributions, which is well known to capture skewed and heavy-tailed distributions often observed in financial data.

Our results show the REIT, TOPIX, and Mothers did not perform well in our sample period by the IRRA and Sharpe ratio. On the other hand, remaining JASDAQ and TSE Second Section perform similarly to the DOW and NASDAQ based on the Sharpe ratio. However, they perform better than the DOW and NASDAQ when evaluated by the IRRA. Therefore, although the JASDAQ and TSE Second Section are markets where not much attention has been paid, their performance is more desirable for risk-averse investors than the world's leading market indexes DOW and NASDAQ. Furthermore, as an additional evidence, we have seen that performance of the JASDAQ standard, a submarket recorded since October 2010, is better than the NASDAQ in terms of the IRRA as well as the Sharpe ratio. Therefore, the JASDAQ standard, a market where not much attention has been paid locally as well as globally, is more desirable than the much publicized vigorous NASDAQ market in terms of the de facto industry standard as well as for risk-averse investors.

Our examination of the stock and REIT markets in Japan in this paper is another exercise of the recently introduced IRRA proposed by Miyahara (2014) which is a value measure based on a risk-averse evaluation. We have provided another evidence of this value measure which can shed a new light for risk-averse investors compared to the Sharpe ratio. Use of the IRRA in addition to the de facto industry standard of the Sharpe ratio can be quite appropriate in evaluation of performance of financial assets.

References

Alexander, C. (2004) Normal mixture diffusion with uncertain volatility: modelling short- and long-term smile effects, *Journal of Banking and Finance*, 28, 2957-2980.

Ban, L., Misawa, T., Miyahara, Y. (2016) Valuation of Hong Kong REIT based on risk sensitive value measure method, *International Journal of Real Options and Strategy*, 4, 1-33.

Brigo, D., Mercurio, F. (2001) Displaced and mixture diffusions for analytically-tractable smile models, Geman, H., Madan, D.B., Pliska, S.R., Vorst, A.C.F. (eds.), *Mathematical Finance, Bachelier Congress 2000*. Berlin: Springer.

Cheridito, P., Delbaen, F., Kupper, M. (2006) Dynamic monetary risk measures for bounded discrete-time processes. *Electronic Journal of Probability*, 11, 57-106.

Cherny, A., Madan, D. (2008) New measures for performance evaluation, *The Review of Financial Studies*, 22, 2571-2606.

Chin, E., Wrigend, A., Zimmermann, H. (1999) Computing portfolio risk using Gaussian mixtures and independent component analysis, *Proceedings of the 1999 IEEE/IAFE/INFORMS, CIFEr'99*, New York, 74-117.

Everitt, B.S., Hand, D.J. (1981) *Finite Mixture Distributions*. New York: Chapman and Hall.

Hastie, T., Tibshirani, R., Friedman, J. (2001) *The Elements of Statistical Learning*, New York: Springer Verlag.

Hodoshima, J., Misawa, T., Miyahara, Y. (2017a) Stock performance by utility indifference pricing and the Sharpe ratio, Nagoya University of Commerce and Business Discussion Paper.

Hodoshima, J., Misawa, T., Miyahara, Y. (2017b) Evaluation of performance of stocks: Evidence from Japan, Nagoya University of Commerce and Business Discussion Paper.

Hodoshima, J., Misawa, T., Miyahara, Y. (2018) Comparison of utility indifference pricing and mean-variance approach under normal mixture, *Finance Research Letters*, 24, 221-229.

JAPAN EXCHANGE GROUP Data & Statistics, Trading by Type of Investors http://www.jpx.co.jp/english/markets/statistics-equities/investor-type/index.html (accessed March 29 2018).

Kon, S.J. (1984) Models of stock returns-a comparison, *Journal of Finance*, 39, 147-165.

Layard, P.R.G., Walters, A.A. (1978) *Microeconomic Theory*, New York: McGraw-Hill.

McLachlan, G., Peel, D. (2000) *Finite Mixture Models: Inference and Applications to Clustering*. Marcel Dekker.

Miyahara, Y. (2010) Risk-sensitive value measure method for projects evaluation, *Journal of Real Options and Strategy*, 3, 185-204.

Miyahara, Y. (2014) Evaluation of the scale risk, *RIMS Kokyuroku*, No. 1886, Financial Modeling and Analysis (2013/11/20-2013/11/22), 181-188.

Miyahara, Y. (2017) [Note] Risk Sensitive Value Measure Methods for Project Evaluations (in Japanese), *Communications of the Japan Association of Real Options and Strategy*,9, 1-95.

Ritchey, R.J. (1990) Call option valuation for discrete normal mixtures, *Journal of Financial Research*, 13, 285-296.

Rolski, T., Schmidli, H., Teugels, J. (1999) *Stochastic Processes for Insurance and Finance*. New York: Wiley.

Titterington, D.M., Smith, A.F.M., Makov, U.F. (1985) *Statistical Analysis of Finite Mixture Distributions*. New York: Wiley.

Zhitlukhin, M. (2014) A second-order modification of the Sharpe ratio, Kijima, M., Muromachi, Y., Shibata, T. (eds.), Recent Advances in Financial Engineering: Proceedings of the TMU Finance Workshop 2014, Singapore: World Scientific Publishing.

Internet Appendix (Proof of Proposition 3) (not for publication)

Lemma *Assume MGF of* $\boldsymbol{X}$ *exists and if the following conditions are satisfied, i.e.,*

$$E[\boldsymbol{X}] > 0 \quad \text{and} \quad P(\boldsymbol{X} < 0) > 0,$$

then $U^{(\alpha)}(\lambda\boldsymbol{X}) > 0$ *for small* $\lambda(> 0)$ *and as* $\lambda \to \infty$, *we have*

$$lim_{\lambda\to\infty} U^{(\alpha)}(\lambda\boldsymbol{X}) = -\infty$$

where $U^{(\alpha)}(\boldsymbol{X}) \equiv -\frac{1}{\alpha} lnE[e^{-\alpha\boldsymbol{X}}]$.

Proof of Lemma. Put $g(\lambda) = U^{(\alpha)}(\lambda\boldsymbol{X})$. Differentiating $g(\lambda)$ with respect to λ, we have

$$g'(\lambda) = -\frac{1}{\alpha}\frac{E[-\alpha X e^{-\alpha\lambda\boldsymbol{X}}]}{E[e^{-\alpha\lambda\boldsymbol{X}}]} = \frac{E[\boldsymbol{X}e^{-\alpha\lambda\boldsymbol{X}}]}{E[e^{-\alpha\lambda\boldsymbol{X}}]}, \quad g'(0) = E[\boldsymbol{X}] > 0.$$

Since $g(0) = U^{(\alpha)}(0) = 0$, this implies $U^{(\alpha)}(\lambda\boldsymbol{X}) > 0$ for small $\lambda(> 0)$.

By the assumption, there exists $a > 0$ and $\delta > 0$ such that $P(\boldsymbol{X} < -a) > \delta$. Then we have

$$E[e^{-\alpha\lambda\boldsymbol{X}}] = E[e^{-\alpha\lambda\boldsymbol{X}} 1_{\boldsymbol{X}<-a}] + E[e^{-\alpha\lambda\boldsymbol{X}} 1_{\boldsymbol{X}\geq -a}] > e^{\alpha\lambda a}\delta.$$

Therefore we have

$$U^{(\alpha)}(\lambda\boldsymbol{X}) = -\frac{1}{\alpha} lnE[e^{-\alpha\lambda\boldsymbol{X}}] < -\frac{1}{\alpha}(\alpha\lambda a + ln\delta) \to -\infty (as \lambda \to \infty).$$

This completes the proof.

Proof of Proposition 3 The IRRA α_0 is the solution of the following equation

$$U^{(\alpha)}(\boldsymbol{X}) = -\frac{1}{\alpha} lnE[e^{-\alpha\boldsymbol{X}}] = 0.$$

We have $U^{(0)}(\boldsymbol{X}) = E[\boldsymbol{X}] > 0$ when $\alpha = 0$. Thus we only need to examine the above equation when $\alpha > 0$ and hence the above equation is equivalent to

$$-lnE[e^{-\alpha\boldsymbol{X}}] = 0.$$

We remark $-lnE[e^{-\alpha\boldsymbol{X}}] = U^{(1)}(\alpha\boldsymbol{X})$. Since $U^{(\alpha)}(\lambda\boldsymbol{X})$ is a concave function of λ (cf., Corollary 2 at page 196 of Miyahara (2010)), $U^{(1)}(\alpha\boldsymbol{X})$ is a concave function of α. By

Lemma given above, it holds that $lim_{\alpha \to \infty} U^{(1)}(\alpha \boldsymbol{X}) = -\infty$, which implies uniqueness of the IRRA. This completes the proof.

Table 1: Summary Statistics of the Four Stock and REIT Indexes Returns

index	mean	s.d.	skewness	kurtosis
TOPIX	0.014	1.539	-0.156	10.068
2nd TSE	0.029	0.932	-1.439	25.204
Mothers	0.038	2.250	-0.703	8.904
JASDAQ	0.032	1.082	-1.078	12.898
REIT	0.009	1.542	-0.092	12.943

Table 2: Bayesian Information Criterion for the Stock and REIT Market Indexes

index	2 components	3 components	4 components
TOPIX	3.535	3.538	NA
2nd TSE	2.308	2.288	2.296
Mothers	4.281	4.284	4.292
JASDAQ	2.721	2.714	2.721
REIT	3.325	3.320	NA

Table 3: Estimates of the Best Fit Normal Mixture Distribution for the Stock and REIT Market Indexes

index	μ_1	μ_2	μ_3	σ_1^2	σ_2^2	σ_3^2	p_2	p_3
TOPIX	0.065	-0.391		1.262	10.857		0.113	
2nd TSE	-0.174	-0.903	0.153	1.144	13.299	0.214	0.026	0.678
Mothers	0.166	-0.707		2.404	19.779		0.147	
JASDAQ	-0.026	-1.016	0.129	1.650	9.862	0.289	0.039	0.626
REIT	0.002	-0.116	0.049	0.458	18.093	2.557	0.070	0.323

Table 4: IRRA in Ascending Order for the Stock and REIT Market Indexes

index	IRRA
REIT	0.007
TOPIX	0.011
Mothers	0.015
JASDAQ	0.054
2nd TSE	0.065

Table 5: Sharpe Ratio in Ascending Order for the Stock and REIT Market Indexes

index	Sharpe Ratio
REIT	0.005
TOPIX	0.009
Mothers	0.017
JASDAQ	0.029
2nd TSE	0.031

Table 6: Summary Statistics of the JASDAQ standard, JASDAQ growth, and NASDAQ

index	mean	s.d.	skewness	kurtosis
JASDAQ standard	0.068	1.064	-1.435	15.613
JASDAQ growth	0.093	2.651	-0.690	9.946
NASDAQ	0.060	1.045	-0.385	6.621

Table 7: Bayesian Information Criterion for the JASDAQ standard, JASDAQ growth, and NASDAQ

index	2 components	3 components	4 components
JASDAQ standard	2.653	2.666	2.662
JASDAQ growth	4.549	4.550	4.560
NASDAQ	2.818	2.819	NA

Table 8: Estimates of the Best Fit Normal Mixture Distribution for the JASDAQ standard, JASDAQ growth, and NASDAQ

index	μ_1	μ_2	σ_1^2	σ_2^2	p_2
JASDAQ standard	-0.230	0.128	4.764	0.385	0.834
JASDAQ growth	0.155	0.078	25.343	2.484	0.801
NASDAQ	0.148	-0.087	0.351	2.303	0.373

Table 9: IRRA in Ascending Order for the JASDAQ standard, JASDAQ growth, and NASDAQ

index	IRRA
JASDAQ growth	0.027
NASDAQ	0.109
JASDAQ standard	0.117

Table 10: Sharpe Ratio in Ascending Order for the JASDAQ standard, JASDAQ growth, and NASDAQ

index	Sharpe Ratio
JASDAQ growth	0.035
NASDAQ	0.057
JASDAQ standard	0.064

Figure 1: Stock price of the TOPIX

Figure 2: Stock price of the Tokyo Stock Exchange Second Section Index

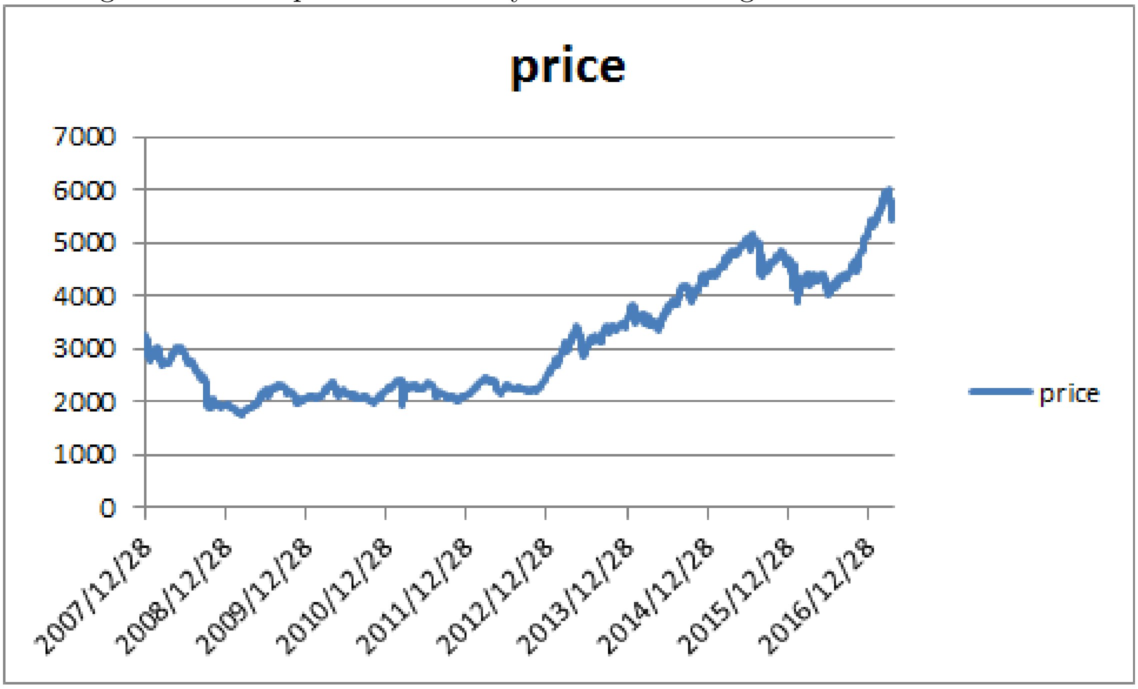

Figure 3: Stock price of the Tokyo Stock Exchange Mothers Index

Figure 4: Stock price of the JASDAQ Index

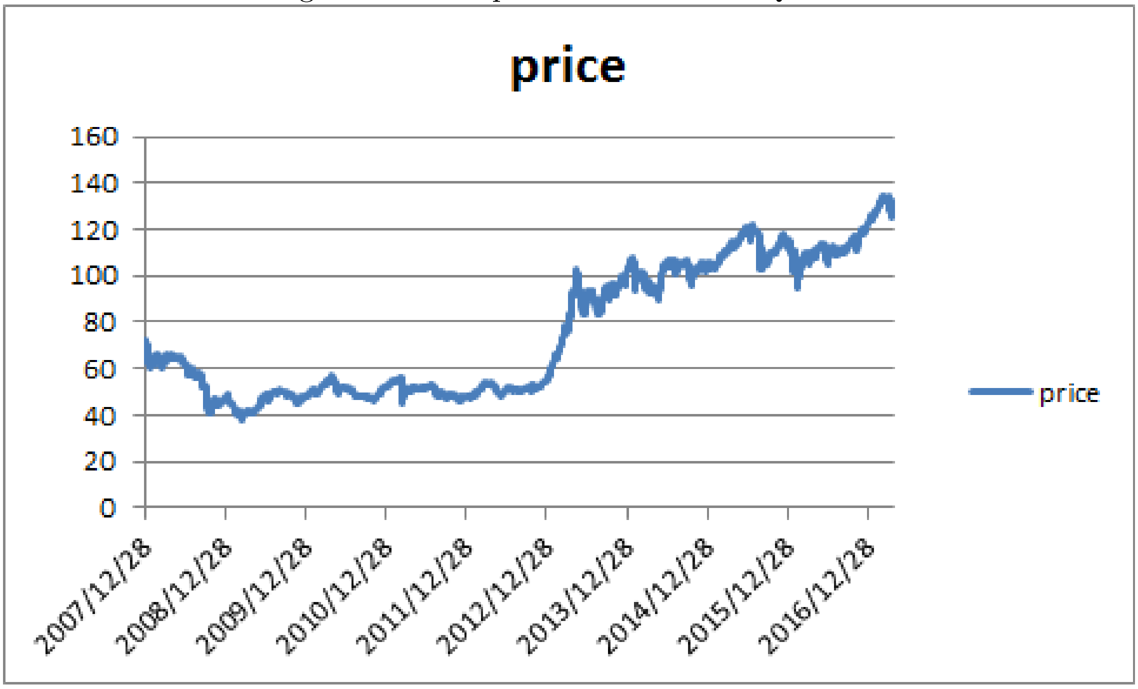

Figure 5: Stock price of the REIT Index

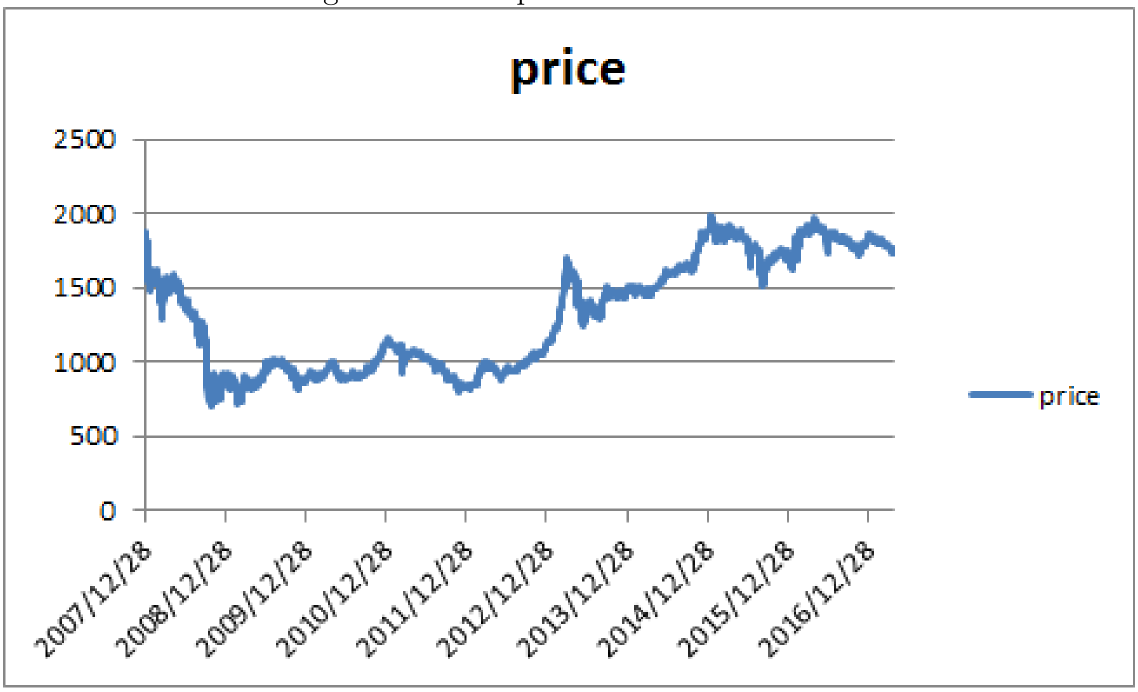

Figure 6: Stock price of the JASDAQ standard

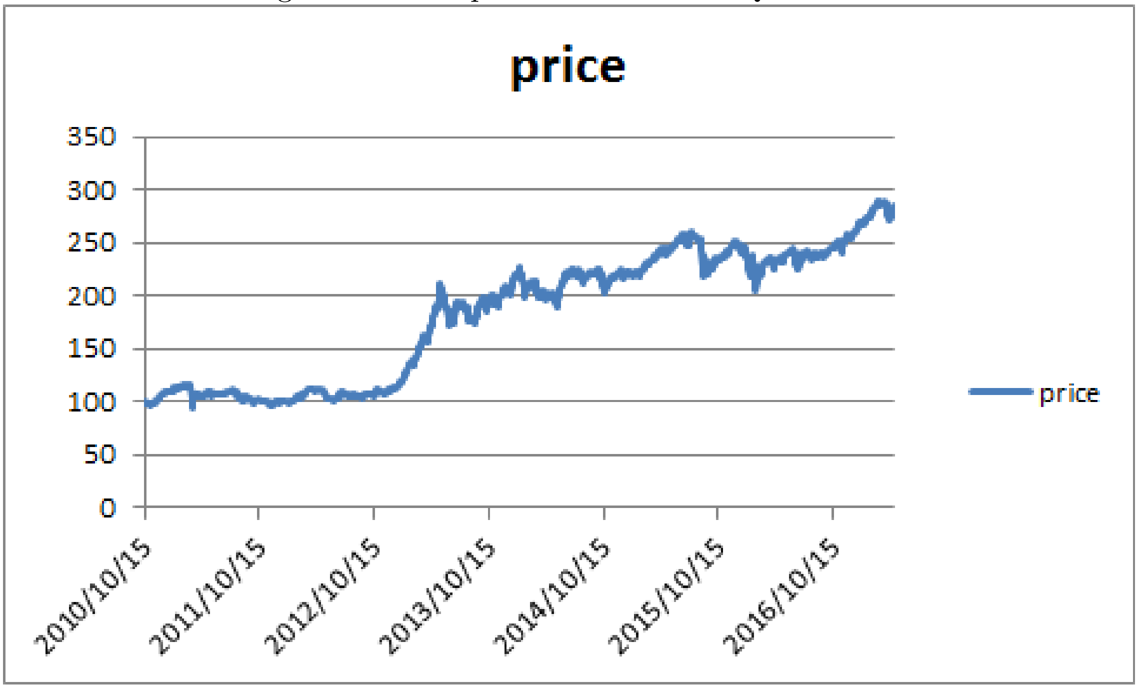

Figure 7: Stock price of the JASDAQ growth

Figure 8: Stock price of the NASDAQ