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Abstract

We consider temporal aggregation of the Aumann-Serrano (AS) and Foster-Hart (FS) performance indexes proposed by Kadan and Liu (2014). The AS performance index is closed under temporal aggregation when the underlying discrete observations are independently and identically distributed and the underlying continuous stochastic process is a Lévy process. The FH performance index is closed under temporal aggregation when the underlying discrete observations are identically distributed and the underlying continuous stochastic process is a stationary process if an approximation of the implicit equation by the Taylor series expansion up to the first order is valid. We present empirical examples using U.S. stock return data.

JEL codes: G11; C22; C46

Keywords: Temporal aggregation; performance index; Aumann-Serrano index; Foster-Hart index

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1 Introduction

Performance measures to evaluate projects, cash flows, assets, etc. are quite important in finance to provide the appropriate assessment. There are many studies of appropriate performance measures. See, e.g., Cogneau and Huber (2009a, 2009b) and Cherny and Madan (2009). Recently, Kadan and Liu (2014) proposed two performance indexes based on the risk indexes proposed by Aumann and Serrano (2008) and Foster and Hart (2009). The concept of the risk index started with Aumann and Serrano (2008) and has received a lot of attention in the financial economics literature (cf., e.g., Foster and Hart, 2009; Hart, 2011; Homm and Pigorsch, 2012; Kadan and Liu, 2014; Schulze, 2014; Niu et al., 2018; Hodoshima and Miyahara, 2020). Hereafter, we call the two performance indexes the AS and FH performance indexes in this paper. The two performance indexes are based on axiomatic approaches, unlike many ad-hoc heuristic approaches replacing traditional performance measures such as the Sharpe ratio (cf., e.g., Eling and Schuhmacher, 2007; Farinelli et al., 2008), and can be quite useful to provide relevant assessment in many financial problems (see empirical examples given in Kadan and Liu, 2014). Although some qualitative properties of the two performance indexes such as Proposition 6 and 7 of Kadan and Liu (201) are presented by Kadan and Liu (2014), there remain some issues about the two performance indexes to be investigated. One of the problems in the AS and FK performance indexes lies in their estimation. Although Kadan and Liu (2014) estimated the two performance indexes by the nonparametric generalized method of moments (GMM) estimator, they did not discuss much estimation. In this paper, we focus on an issue of temporal aggregation of the AS and FK performance indexes. A model can be specified for an observable frequency by assuming it is the correct model for this frequency. We say a model is closed under temporal aggregation if the model keeps the same parameter value for any data frequency. Since the two performance indexes are both univariate performance measures, their observations are naturally time series observations. Then, temporal aggregation becomes an issue, and it is an unexplored research topic for the AS and FH performance indexes. It is beneficial to applied researchers to guide on the issue of temporal aggregation when they estimate the AS and FH performance indexes with daily, weekly, and monthly observations. With respect to the *de facto* industry-standard performance measure Sharpe ratio, there exist studies of temporal aggregation such as Lo (2002), Opdyke (2007), and Ledoit and Wolf (2008). Temporal aggregation of a performance index is an issue on the performance index when the frequency of observations changes from high to low and vice versa. A well-known paper of temporal aggregation in econometrics is Drost and Nijman (1993) who studied temporal aggregation on the autoregressive conditional heteroskedastic (ARCH) model of Engle (1982) and the generalized autoregressive conditional heteroskedastic (GARCH) model of Bollerslev (1986). There also exist other studies of temporal aggregation on volatility models such as Meddahi and Renault (2004).

In order to study temporal aggregation on the AS and FK performance indexes, we focus on their defining implicit equations. The AS performance index of a gamble $\boldsymbol{g}$ is defined to be a positive solution α of the implicit equation given by

$$E[\exp(-\alpha\boldsymbol{g})] = 1,$$

where a gamble $\boldsymbol{g}$ denotes a random variable such as assets, cash flows, projects, etc. The above equation is equal to equation (1) of Kadan and Liu (2014). On the other hand, the FH performance index of a gamble $\boldsymbol{g}$ is defined to be a positive solution γ of the implicit equation

$$E[\ln(1 + \gamma\boldsymbol{g})] = 0.$$

The above equation is equal to equation (3) of Kadan and Liu (2014). The AS and FH risk indexes are the reciprocals of the AS and FH performance indexes, i.e., the AS risk index proposed by Aumann and Serrano (2008) is given by $1/\alpha$ and the FH risk index proposed by Foster and Hart (2011) is given by $1/\gamma$. Therefore, the two risk indexes correspond to the two performance indexes one to one, and hence temporal aggregation on the two performance indexes is equivalent to temporal aggregation on the two risk indexes. The AS and FK performance indexes are static performance measures of one-period gambles[1] $\boldsymbol{g}$. Although the underlying gamble of the two performance indexes is a static random variable, its observations are time series observations. Then, we need to consider time

[1]Kadan and Liu (2014) extended the two performance indexes to multi-period gambles. However, temporal aggregation of the two performance indexes for multi-period gambles is beyond the scope of the present study.

series properties of the underlying random variable to deal with temporal aggregation of the two performance indexes. In Section 2, we consider temporal aggregation of the AS and FH performance indexes given as the solutions of the implicit equations given above. In Section 3, we provide empirical examples of the AS and FH performance indexes using U.S. stock data of daily, weekly, and monthly observations in the same sample period along with the Sharpe ratio. In our empirical examples, the two performance indexes take similar values in daily, weekly, and monthly observations. There exist some cases where the two performance indexes show time-invariant properties under temporal aggregation, i.e., the two performance indexes are closed or nearly closed under temporal aggregation, in our empirical examples. On the other hand, the Sharpe ratio scores uniformly increase as the frequency of observations becomes less from daily to weekly and monthly.

The rest of the paper is organized as follows. In Section 2, we present sufficient conditions for the AS and FH performance indexes to be closed under temporal aggregation. In Section 3, we provide empirical examples of the two performance measures and the Sharpe ratio using daily, weekly, and monthly observations of U.S. stock data in the same sample period. In Section 4, we present concluding comments.

2 Temporal aggregation of the AS and FH performance indexes

In this section, we consider temporal aggregation of the AS and FH performance indexes considered by Kadan and Liu (2014).

We first consider the AS performance index. Suppose the AS performance index of a gamble $\boldsymbol{g}_t$ at time t is given by α, which is a unique solution α of the implicit equation

$$E[\exp(-\alpha \boldsymbol{g}_t)] = 1. \tag{1}$$

Let us consider the following sequence of random variables $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ where $\boldsymbol{g}_i (i = t, \cdots, t+q-1)$ denotes a random variable of a gamble at time i. The implicit equation of the AS performance index for the sum of random variables $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ is given by

$$E[\exp(-\alpha(\boldsymbol{g}_t + \cdots + \boldsymbol{g}_{t+q-1}))] = 1 \tag{2}$$

where α denotes the AS performance index of the sum of random variables $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$. When each gamble of $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ is independent each other, the left hand side of equation (2) is given by

$$E[\exp(-\alpha \boldsymbol{g}_t)] \cdots E[\exp(-\alpha \boldsymbol{g}_{t+q-1})] = 1. \tag{3}$$

If each gamble of $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ is identically distributed, then we have the following equality

$$E[\exp(-\alpha(\boldsymbol{g}_t + \cdots + \boldsymbol{g}_{t+q-1}))] = E[\exp(-\alpha \boldsymbol{g}_t)]^q = 1, \tag{4}$$

which implies the same implicit equation as equation (1) given above, which in turn implies the solution α of the implicit equation (2) is the same as the solution α of the implicit equation (1). Therefore, when the sequence of random variables $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ is independently and identically distributed (i.i.d.), the AS performance index is closed under temporal-aggregation.

The above case applies to when observations of the random variable of a gamble are discrete time series. If the stochastic process of a gamble is a continuous process, the AS performance index becomes closed under temporal aggregation when the underlying continuous process is a Lévy process where the characteristic function $E[e^{-iu\boldsymbol{g}_t}](\equiv \phi_t(u))$ of the stochastic process $\boldsymbol{g}_t$ has the following property

$$\phi_t(u) = (\phi_1(u))^t. \tag{5}$$

A Lévy process is a continuous-time analogue of a random walk. See, e.g., Barndorff-Nielsen et al. (2001) and Sato (1999) for references. Suppose the moment-generating function (MGF) of the stochastic process $\boldsymbol{g}_t$ exists for any t. We remark the left-hand side of the implicit equation (1) has , besides the minus sign, the form of the MGF of a gamble $\boldsymbol{g}_t$. Then, equation (5)[2] implies the AS performance index is closed under temporal aggregation since a relationship of the characteristic function implies the same relationship of the MGF if the MGF exists. We now summarize the result as follows.

[2]Similarly to equation (5), we have different equalities of the characteristic equation of a Lévy process $\boldsymbol{g}_t$ such as

$$\phi_t(u) = (\phi_s(u))^{t/s}, \quad 0 \le t \le T, \quad 0 < s \le T.$$

where T denotes an endpoint of time.

Theorem 1 *1. When observations of the random variable of a gamble are discrete time series, the AS performance index is closed under temporal aggregation if the sequence of the random variable of a gamble is i.i.d.*

2. When the stochastic process of a gamble is a continuous process, the AS performance index is closed under temporal aggregation if the underlying continuous process is a Lévy process.

We then consider the FH performance index. The FH performance index of a gamble $\boldsymbol{g}_t$ is defined by γ which is the unique solution of the implicit equation

$$E[\ln(1+\gamma \boldsymbol{g}_t)] = 0. \tag{6}$$

Let us consider the following sequence of random variables $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ where $\boldsymbol{g}_i (i = t, \cdots, t+q-1)$ denotes a random variable of a gamble at time i. The implicit equation of the FH performance index for the sum of random variables $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ is given by

$$E[\ln(1+\gamma(\boldsymbol{g}_t + \cdots + \boldsymbol{g}_{t+q-1}))] = 0. \tag{7}$$

By a stochastic expansion of $\ln(1+\gamma(\boldsymbol{g}_t+\cdots+\boldsymbol{g}_{t+q-1}))$ and $\ln(1+\gamma\boldsymbol{g}_t)+\cdots+\ln(1+\gamma\boldsymbol{g}_{t+q-1})$ with respect to $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ up to the first order, we have the following equality

$$\ln(1+\gamma(\boldsymbol{g}_t+\cdots+\boldsymbol{g}_{t+q-1})) = \ln(1+\gamma\boldsymbol{g}_t)+\cdots+\ln(1+\gamma\boldsymbol{g}_{t+q-1}) = \gamma(\boldsymbol{g}_t+\cdots+\boldsymbol{g}_{t+q-1}) \tag{8}$$

where a stochastic expansion of $f(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_m)$ denotes a Taylor series expansion of a function f with respect to random variables $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_m$. If each of random variables $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$ is identically distributed and γ is the solution of the implicit equation

$$E[\ln(1+\gamma\boldsymbol{g}_i)] = 0 (i = t, \cdots, t+q-1), \tag{9}$$

then we have the following equality

$$E[\ln(1+\gamma(\boldsymbol{g}_t+\cdots+\boldsymbol{g}_{t+q-1}))] = E[\ln(1+\gamma\boldsymbol{g}_t)]+\cdots+E[\ln(1+\gamma\boldsymbol{g}_{t+q-1})] = 0. \tag{10}$$

This implies the FH performance index γ of each gamble $\boldsymbol{g}_i (i = t, \cdots, t+q-1)$ continues to be the FH performance index of the sum of gambles $\boldsymbol{g}_t, \cdots, \boldsymbol{g}_{t+q-1}$. In other words, the FH performance index is closed under temporal aggregation. This holds when each gamble is identically distributed and the Taylor series approximation up to the first order is valid. This can be extended to a continuous process where the FH performance index is closed under temporal aggregation when the underlying continuous process is a stationary process and the Taylor series approximation up to the first order is valid. We now summarize the result as follows.

Theorem 2 *1. When observations of the random variable of a gamble are discrete time series, the FH performance index is closed under temporal aggregation if the sequence of the random variable of a gamble is identically distributed and the Taylor series approximation of the implicit equation up to the first order is valid.*

2. When the stochastic process of a gamble is a continuous process, the FH performance index is closed under temporal aggregation if the underlying continuous process is a stationary process and the Taylor series approximation of the implicit equation up to the first order is valid.

We take expectation of the Taylor series approximation of the implicit equation up to the first order in the above argument when we try to solve for the FH performance index. How much expectation of the Taylor series approximation of the implicit equation up to the first order approximates expectation of the implicit equation depends on the underlying distribution of the random variable of a gamble or the underlying stochastic process of a gamble. We examine in the next section how much the temporal aggregation theorems of the AS and FH performance indexes given above hold in real empirical examples of U.S. stock return data.

3 Empirical Examples

In this section, we present empirical examples of the AS and FH performance indexes using a selection of U.S. stock return data. As return data, we use daily, weekly, and monthly return data[3] from January 2008 till April 2017, the period including the global financial crisis, in order to examine the effect of temporal aggregation on the AS and FH

[3]Data available on request from the author.

performance indexes. We employ a log-return g_t as a stock return defined by

$$g_t = 100(\ln(P_t) - \ln(P_{t-1}))$$

where g_t is a log-return at time t, and P_t and P_{t-1} are respectively the price of a stock at time t and $t-1$. This is to make time aggregation relevant, e.g., to make the sum of five daily returns a weekly return, i.e.,

$$g_t + g_{t-1} + g_{t-2} + g_{t-3} + g_{t-4} = 100(ln(P_t) - ln(P_{t-5}))$$

where we assume returns on Saturday and Sunday are not available so that the sum of five daily returns usually makes a weekly return.

We use, as a selection of U.S. stocks, the two stock market indexes of Dow Jones Industrial Average (DOW) and S&P500 and individual stocks of McDonald, Boeing, and UnitedHealth.

We first present summary statistics of a selection of stocks for daily, weekly, and monthly return data at Tables 1, 2, and 3.

Table 1 provides summary statistics of daily return data for a selection of U.S. stocks. UH stands for UnitedHealth. Mean ranges from 0.019 in DOW to 0.052 in UnitedHealth. Standard deviation ranges from 1.188 in McDonald to 2.201 in UnitedHealth. Thus UnitedHealth is a stock with the highest mean and the highest volatility. The two stock indexes are negatively skewed. But there are individual stocks with positive skewness, i.e., McDonald and UnitedHealth, as well as with a stock with negative skewness, i.e., Boeing. All the stocks have heavy tails compared to the normal distribution. In particular, UnitedHealth has the highest kurtosis.

Table 2 provides summary statistics of weekly return data for a selection of U.S. stocks. Mean ranges from 0.102 in DOW to 0.258 in UnitedHealth. Weekly mean is nearly five times as much as daily mean in all the stocks. This should be the case when daily returns are identically distributed. Standard deviation ranges from 2.343 in McDonald to 5.006 in UnitedHealth. Thus, UnitedHealth is a stock with the highest mean and the highest volatility in weekly data too. When daily returns are i.i.d., standard deviation of weekly returns is close to $\sqrt{5} = 2.236$ times as much as standard deviation of daily returns. However, standard deviation of weekly returns is a little smaller than

$\sqrt{5}$ times as much as standard deviation of daily returns. For example, in DOW weekly standard deviation is 2.453, which is a little smaller than $2.728 = \sqrt{5} \times 1.220$, where 1.220 is daily standard deviation. In weekly data, all the stocks are negatively skewed. All the stocks have heavy tails compared to the normal distribution with the maximum in UnitedHealth.

Table 3 provides summary statistics of monthly return data. Mean ranges from 0.408 in DOW to 1.086 in UnitedHealth. If daily returns are identically distributed, then mean of monthly returns should be close to $30 \div 7 \times 5$ times as much as mean of daily returns, where 30 denotes 30 days in a month and 7 denotes the number of days in a week. Mean of monthly returns is not much different from the above formula computed from mean of daily returns. Standard deviation ranges from 4.066 in McDonald to 8.224 in UnitedHealth. UnitedHealth is a stock with the highest mean and the highest volatility in monthly data too. If daily returns are i.i.d., then standard deviation of monthly returns should be close to $\sqrt{30 \div 7 \times 5}$ times as much as that of daily returns. However, standard deviation of monthly returns is smaller than the above formula. For example, in DOW standard deviation of monthly returns is 4.245, which is less than 5.647, which is the number obtained from the above formula. Again all the stocks are negatively skewed in monthly data. All the stocks have heavy tails compared to the normal distribution in monthly data too. However, they all have tails thinner than tails in daily and weekly data.

We then show the AS and FH performance indexes as well as the Sharpe ratio of a selection of stocks for daily, weekly, and monthly data. We estimate the two performance measures by the generalized method of moments (GMM) estimator as described in Kadan and Liu (2014). The GMM estimator is consistent and asymptotically normally distributed. Table 4 provides the GMM estimates of the three performance measures for daily, weekly, and monthly data. The GMM estimates of the AS and FH performance indexes are obtained using the sample analogs of the two implicit equations with grid search to search for the indexes where the sample analogs of the two implicit equations denote the sample method of moments estimates of the two implicit equations. The GMM estimate of the Sharpe ratio is obtained using the sample analog from data.

We can see the AS performance index is almost the same as the FH performance index in daily return data at Table 4. The AS and FH performance indexes are similar in all the stocks except McDonald with scores about three times as much as those in other stocks. On the other hand, the Sharpe ratio is smaller than the AS and FH performance indexes except for UnitedHealth with a slightly higher Sharpe ratio than the two performance indexes.

The AS and FH performance indexes in weekly data are a little higher than those in daily data in DOW, S&P500, and McDonald while the opposite is the case in Boeing and UnitedHealth. On the other hand, the Sharpe ratio is uniformly larger in weekly data than in daily data. If daily returns are i.i.d., then the Sharpe ratio of weekly returns is close to $\sqrt{5} = 2.236$ times as much as that of daily returns (cf. Lo, 2002). The Sharpe ratio scores are higher than the scores by this rule in DOW, S&P500, and McDonald. For example, in DOW the weekly Sharpe ratio score 0.039 is higher than 0.034, the score by the above rule. On the other hand, the opposite is the case in Boeing and UnitedHealth. In UnitedHealth, the weekly Sharpe ratio score 0.050 is slightly smaller than 0.051, the score by the above rule.

The AS and FH performance indexes are uniformly higher in monthly data than those in daily and weekly data. The increase in the two performance indexes in monthly data is substantial in DOW, S&P500, and McDonald but small in Boeing and UnitedHealth. However, the increase in the two performance indexes in monthly data is small compared to that in the Sharpe ratio in monthly data compared to daily and weekly data. When daily returns are i.i.d., the monthly Sharpe ratio is close to $4.629 (= \sqrt{30 \div 7 \times 5})$ times as much as the daily Sharpe ratio. The monthly Sharpe ratio is considerably higher than 4.629 times as much as the daily Sharpe ratio except for Boeing.

Overall, the AS and FH performance indexes increase as the frequency of observations decreases from daily to weekly and monthly in DOW, S&P500, and McDonald but tend to remain time-invariant in individual stocks of Boeing and UnitedHealth. The AS and FH performance indexes are much more stable than the Shape ratio as the frequency of observations changes from daily to weekly and monthly. Therefore, in our empirical examples, there are two stocks, Boeing and UnitedHealth, which are nearly closed under temporal aggregation. We also observed the two performance measures are much more stable than the Sharpe ratio when the observation frequency decreases from daily to weekly and monthly.

4 Concluding comments

We have studied temporal aggregation of the AS and FH performance indexes recently proposed by Kadan and Liu (2014) in this study. The AS performance index is shown to be closed under temporal aggregation if the underlying discrete observations are i.i.d and the underlying continuous process is a Lévy process. On the other hand, the FH performance index is shown to be closed under temporal aggregation when the underlying discrete observations are identically distributed and the underlying continuous process is a stationary process if the approximation of the implicit equation of the FH performance index is valid by the first order Taylor series expansion. In empirical examples, we have computed the two performance indexes along with the Sharpe ratio using daily, weekly, and monthly return data for a selection of U.S. stocks. We have found time-invariant properties of the two performance indexes in some stocks. Furthermore, the two performance indexes are much more stable compared to the Sharpe ratio when the frequency of observations changes from daily to weekly and monthly.

5 Declarations of Interests: none

6 Data available on request from the author.

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Table 1: Summary Statistics of Daily Return Data for a Selection of Stocks

name	mean	s.d.	skewness	kurtosis
DOW	0.019	1.220	-0.071	13.197
S&P500	0.021	1.329	-0.324	13.330
McDonald	0.050	1.188	0.095	9.559
Boeing	0.043	1.847	-0.004	7.934
UH	0.052	2.201	0.463	28.260

In the table, s.d. denotes standard deviation. UH stands for UnitedHealth.

Table 2: Summary Statistics of Weekly Return Data for a Selection of Stocks

name	mean	s.d.	skewness	kurtosis
DOW	0.102	2.453	-1.025	13.240
S&P500	0.108	2.623	-0.967	11.747
McDonald	0.247	2.343	-0.191	5.508
Boeing	0.210	4.244	-0.560	7.144
UH	0.258	5.006	-0.479	17.360

In the table, s.d. denotes standard deviation. UH stands for UnitedHealth.

Table 3: Summary Statistics of Monthly Return Data for a Selection of Stocks

name	mean	s.d.	skewness	kurtosis
DOW	0.408	4.245	-0.819	4.444
S&P500	0.433	4.542	-0.945	5.072
McDonald	1.043	4.066	-0.221	3.449
Boeing	0.893	7.924	-0.950	4.602
UH	1.086	8.224	-1.628	8.328

In the table, s.d. denotes standard deviation. UH stands for UnitedHealth.

Table 4: The GMM Estimates of the AS, FH, and Sharpe ratio for a Selection of Stocks

	name	AS	FH	Sharpe
daily data	DOW	0.026	0.026	0.015
	S&P500	0.023	0.023	0.015
	McDonald	0.070	0.069	0.041
	Boeing	0.025	0.025	0.023
	UH	0.021	0.021	0.023
weekly data	DOW	0.033	0.031	0.039
	S&P500	0.031	0.029	0.039
	McDonald	0.088	0.076	0.103
	Boeing	0.023	0.022	0.048
	UH	0.020	0.018	0.050
monthly data	DOW	0.043	0.039	0.091
	S&P500	0.040	0.035	0.090
	McDonald	0.121	0.092	0.250
	Boeing	0.027	0.023	0.110
	UH	0.028	0.022	0.129

UH stands for UnitedHealth.