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Abstract

In this study, we consider the mean-variance utility maximization problem for banks. Especially, we consider the utility maximization problems of the bank's portfolio return and accounting profit. Moreover, we consider both balance sheet models irrespective of whether the items on the liability side of the balance sheets are internalized in terms of assets. The calibration result shows that the accuracy of the model fitting for the utility maximization model without internalizing the balance sheet is the most inferior to the models with the internalized balance sheet model. Moreover, we observe that there is no significant difference in the accuracies of the model fitting of the utility maximization models for the portfolio return and accounting profit as long as the balance sheet is internalized. To practically describe the bank's behavior, the internalization of the balance sheet model is more important than the portfolio return or accounting profit for which the bank maximizes the mean-variance utility.

JEL Classification: G11, G21
Keywords: bank's behavior, bank's asset allocation, mean-variance analysis

1 Introduction

We consider the classical optimal asset allocation problem for a bank. We suppose that the bank's preference for risk is presented by the mean-variance utility, and the bank seeks an optimal loan ratio to maximize the expected utility. The loan ratio in this study is measured by the proportion of the amount of lending to the total (risky) asset, which is the sum of the amounts of lending and investing in securities.

The mean-variance analysis by Markowitz (1952) has been traditionally used to analyze a bank's behavior in asset allocation. Kane and Malkiel (1965), Kahane (1977), Koehn and Santomero (1980), Kim and Santomero (1988), Greenwald and Stiglitz (1989), and Keeley and Frederick (1990) consider the mean-variance utility maximization problem for the return of the bank's asset portfolio. In contrast, Ishii (1971) and Halaj (2013) consider the mean-variance utility maximization problem for the bank's accounting profit calculated by the difference of the portfolio return and the sum of funding costs. Thus, in the mean-variance framework, there are two types of models in which the agent maximizes its utility for the portfolio return and accounting profit. Besides the mean-variance utility maximization problem, in fact, Fischer (1983), Hartley and Walsh (1991), Jacques

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(2008), and Wang (2013) also consider the bank's optimization problems for the accounting profit. Moreover, Ishii (1971) internalizes the items on the liability side of the balance sheets in terms of assets, while most previous studies treat all items on the liability side as exogenous variables. Not only does this enable the model to contain more variables and parameters but also leads to the optimal management of the bank's funding.

Most previous studies focus on the effects of the central bank's financial policy on the macroeconomy or of the bank regulations on bank behavior (i.e., lending and funding). However, few studies examine the fit of the theoretical model of the bank's behavior with its actual behavior. This study addresses this problem. We verify whether the maximization problem for the portfolio return or accounting profit more closely describe the bank's actual behavior.

We consider the bank's mean-variance utility maximization problems for both the portfolio return and accounting profit. We also consider two balance sheet models for the bank, whether the items on the liability side of the balance sheet are internalized in terms of assets. We construct the internalized balance sheet model following Ishii (1971). Thus, we treat three optimization problems for the bank: (i)its mean-variance utility maximization problem for the portfolio return without the internalized balance sheet model, (ii)its mean-variance utility maximization problem for the portfolio return with the internalized balance sheet model, and (iii)its mean-variance utility maximization problem for the accounting profit with the internalized balance sheet model. We then solve an optimal loan ratio for each optimization problem.

After obtaining the optimal loan ratios, we calibrate the model parameters to fit the model loan ratio to the actual one. This simultaneously verifies the accuracy of the model fit to the actual data. For the maximization problem for the portfolio return, we use only the data for the balance sheet. In contrast, for the maximization problem of the accounting profit, we use data from both the balance sheet and profit and loss statements. As examples, we use the financial statements of five large Japanese banking companies. Moreover, we perform two types of calibration. We implement the first under the assumption that all parameters to be estimated are common across the five banks (cross-bank data). The other is to calibrate the models using the historical data of a bank's financial statement (historical data).

The results are as follows: at first, the accuracy of the model fitting for the model maximizing the utility for the portfolio return without the internalized balance sheet model is the most inferior for both cases of cross-bank data and historical data. We believe that this result is straightforward, as the model without the internalized balance sheet model has the least parameters and variables. Next, when using cross-bank data, the model optimizing the utility for the accounting profit with the internalized balance sheet model has the most accurate fit. However, the estimation errors for the model maximizing the utilities for the portfolio return and accounting profit with the internalized balance sheet model are quite close. Finally, when using historical data, the model optimizing the utility for the portfolio return with the internalized balance sheet model is surprisingly the most accurate in fit. As the model optimizing the utility for the accounting profit takes into account more variables than for the portfolio return, it is natural to expect that it has superior accuracy of fit than the model optimizing the utility for the portfolio return. Therefore, our results show that reducing exogenous variables for modeling financial statements is more important than adding parameters and variables to the model in the context of the mean-variance framework, for modeling the actual behavior of banks.

2 Model

2.1 Notations

The notations in our study follow Ishii (1971).

- L: money amount of lending
- B: money amount invested into securities
- D_p: intrinsic deposit, constant and given
- D: secondary deposit
- D_g: total deposit defined by $D_g = D_p + D$
- N: borrowed money at interbank markets
- C_r: cash on the asset side of the balance sheet
- r: proportion of cash to the total deposit, that is, $r = \frac{C_r}{D_g}$
- l: loan ratio defined as the proportion of the money amount of lending to the total (risky) asset, that is, $l = \frac{L}{L+B}$
- e: capital and liabilities with long maturities, constant and given
- C_D: cost of deposits
- C_N: cost of borrowing money
- C_e: cost of equity (funding cost to shareholders), constant and given
- C: total cost of funding defined by $C = C_D + D_N + C_e$

2.2 Balance Sheet Models

2.2.1 Simple Balance Sheet Model

We first introduce a simple balance sheet model where all items on the liability side of the balance sheet are exogenously given.

The bank's balance sheet satisfies

$$C_r + L + B = D_g + N + e. \tag{2.1}$$

Substituting the definition of r into (2.1), the balance sheet model (2.1) is

$$L + B = X, \tag{2.2}$$

where $X := (1 - r)D_g + N + e$.

Hence, the definition of l and (2.2) give

$$L = lX \tag{2.3}$$

and

$$B = (1 - l)X. \tag{2.4}$$

2.2.2 Internalized Balance Sheet Model

Next, we introduce an internalized balance sheet model where the items on the liability side of the balance sheet are presented in terms of the assets (Ishii 1971).

The secondary deposit D is defined by

$$D = k_1 L + k_2 B,$$

where k_i $(i = 1, 2)$ is the extraction rate and a constant. The borrowed money at interbank markets N is given as a function of the loan L,

$$N = \lambda L,$$

where $\lambda \geq 0$ is a constant.

From (2.1), and the definitions of D, D_g and N, we have

$$\left(\frac{1}{b} - \frac{\lambda}{1-r}\right) L + \frac{1}{a} B = D_p + e', \tag{2.5}$$

where

$$a = \frac{1-r}{1-(1-r)k_2},$$
$$b = \frac{1-r}{1-(1-r)k_1},$$

and

$$e' = \frac{e}{1-r}.$$

From (2.5) and the definition of l, it holds

$$X = \frac{M_2}{b - M_1 l}, \tag{2.6}$$

where

$$M_1 = b - a + c\lambda,$$
$$M_2 = ab(D_p + e'),$$

with

$$c = \frac{ab}{1-r}.$$

Therefore, from (2.4), the internalized balance sheet model shows that the money amount of securities B is given by

$$B = \frac{M_2(1-l)}{b - M_1 l}, \tag{2.7}$$

and the loan L is given by

$$L = \frac{M_2 l}{b - M_1 l}. \tag{2.8}$$

2.3 Stochastic Model for Assets

We denote the gross return per unit of lending by R_L and the gross return per unit of security holdings by R_B. We suppose that R_j $(j = L, B)$ is a random variable and denote the expectation, variance as E_j and σ_j^2 $(j = L, B)$, respectively. Moreover, the correlation coefficient of R_L and R_B is denoted by ρ_{LB}. We also assume $R_j \geq 1$ $(j = L, B)$ without loss of generality.

2.4 Preference of the Bank

We suppose that the bank has a mean-variance utility $U(V)$ for a random economic variable V,

$$U(V) = E[V] - \frac{1}{2}KVar[V],$$

where K is the risk-aversion parameter.

3 Optimal Asset Allocation under Simple Balance Sheet Model

In this section, we consider the bank's utility maximization problem for the return of the asset portfolio, when the deposits and borrowed money are exogenously given, that is, the model given in Section 2.2.1.

From (2.3) and (2.4), the gross return R_g for the bank's asset portfolio is

$$\begin{aligned} R_g =& R_L L + R_B B \\ =& (R_L - R_B)Xl + R_B X, \end{aligned}$$

where $X = (1-r)D_g + N + e$. Then, the expected gross profit E_g is

$$E_g := E[R_g] = ((\mu_L - \mu_B)Xl + \mu_B X),$$

and the variance of R_g is

$$\sigma_g^2 := Var[R_g] = (\sigma_L^2 + \sigma_B^2 - 2\rho_{LB}\sigma_L\sigma_B)X^2l^2 + 2(\rho_{LB}\sigma_L\sigma_B - \sigma_B^2)X^2l + \sigma_B^2X^2.$$

The bank chooses the loan ratio l to maximize the utility U for the gross return R_g of the asset portfolio, that is,

$$\max_l E_g - \frac{1}{2}K\sigma_g^2.$$

From the first-order-condition for the optimization, we solve

$$\frac{\partial E_g}{\partial l} - \frac{1}{2}K\frac{\partial \sigma_g^2}{\partial l} = 0.$$

This leads to

$$\zeta_1 l - \zeta_2 = 0,$$

where

$$\begin{aligned} \zeta_1 =& K(\sigma_L^2 + \sigma_B^2 - 2\rho_{LB}\sigma_L\sigma_B)X, \\ \zeta_2 =& \mu_L - \mu_B - K(\rho_{LB}\sigma_L\sigma_B - \sigma_B^2). \end{aligned}$$

Hence, the optimal loan ratio l^* is

$$l^* = \frac{\zeta_2}{\zeta_1}. \qquad (3.1)$$

4 Optimal Asset Allocations under Internalized Balance Sheet Model

4.1 Optimal Asset Allocation for Portfolio Return (Non-P/L)

In this section, we consider the bank's utility maximization problem for the return of the asset portfolio, when the intrinsic deposit is exogenously given, that is, the model in Section 2.2.2. We do not incorporate into the profit and loss statement at this stage yet.

From (2.7) and (2.8), the gross return R_g for the bank's asset portfolio is

$$\begin{aligned} R_g =& R_L L + R_B B \\ =& R_L \frac{M_2 l}{b - M_1 l} + R_B \frac{M_2(1-l)}{b - M_1 l} \\ =& \frac{M_2}{b - M_1 l}((R_L - R_B)l + R_B). \end{aligned}$$

Then, the expected gross return E_g is

$$E_g := E[R_g] = \frac{M_2}{b - M_1 l}((\mu_L - \mu_B)l + \mu_B),$$

and the variance of R_g is

$$\sigma_g^2 := Var[R_g] = \left(\frac{M_2}{b - M_1 l}\right)^2 ((\sigma_L^2 + \sigma_B^2 - 2\rho_{LB}\sigma_L\sigma_B)l^2 + 2(\rho_{LB}\sigma_L\sigma_B - \sigma_B^2)l + \sigma_B^2).$$

The bank's optimization problem is to find the optimal loan ratio l^* to maximize the expected utility, that is,

$$\max_l E_g - \frac{1}{2}K\sigma_g^2.$$

From the first-order-condition for the optimization, we solve

$$\frac{\partial E_g}{\partial l} - \frac{1}{2}K\frac{\partial \sigma_g^2}{\partial l} = 0.$$

This leads to

$$\zeta_3 l - \zeta_4 = 0,$$

where

$$\begin{aligned} \zeta_3 =& KM_2(b(\sigma_L^2 + \sigma_B^2 - 2\rho_{LB}\sigma_L\sigma_B) + M_1(\rho_{LB}\sigma_L\sigma_B - \sigma_B^2)) + M_1((\mu_L - \mu_B)b + \mu_B M_1), \\ \zeta_4 =& b((\mu_L - \mu_B)b + \mu_B M_1) - KM_2((\rho_{LB}\sigma_L\sigma_B - \sigma_B^2)b + \sigma_B^2 M_1). \end{aligned}$$

Then, the optimal loan ratio l^* is

$$l^* = \frac{\zeta_4}{\zeta_3}. \tag{4.1}$$

4.2 Optimal Asset Allocation for Accounting Profit (P/L)

In this section, we consider the bank's utility maximization problem for the accounting profit, when the intrinsic deposit is exogenously given, that is, the model in Section 2.2.2. For this, we fist define the funding costs according to Ishii (1971). The costs of deposit and borrowing money are proportional to the amounts of deposit and borrowing money,

$$C_D = \eta D_g, \quad C_N = \gamma N (= \gamma \lambda L),$$

where η and γ are constants.

The accounting profit R_n for the bank is defined by the difference of the portfolio return R_g and the total funding cost C, that is,

$$R_n = R_g - C.$$

From (2.7), (2.8), and the definitions of C_D and C_N, the accounting profit R_n is rewritten as

$$\begin{aligned} R_n =& R_g - C \\ =& R_L L + R_B B - (C_D + C_N + C_e) \\ =& \frac{M_2}{b - M_1 l}((R_L - R_B)l + R_B) - \eta \left(D_p + \frac{(k_1 M_1 - k_2 M_2)l + k_2 M_2}{b - M_1 l} \right) + \gamma\lambda \frac{M_2 l}{b - M_1 l} + C_e. \end{aligned}$$

Then, the expectation of the accounting profit E_n is

$$E_n := E[R_n] = \frac{M_2}{b - M_1 l}((\mu_L - \mu_B)l + \mu_B) - \eta \left(D_p + \frac{(k_1 M_1 - k_2 M_2)l + k_2 M_2}{b - M_1 l} \right) + \gamma\lambda \frac{M_2 l}{b - M_1 l} + C_e,$$

and the variance of R_n is

$$\begin{aligned} \sigma_n^2 :=& Var[R_n] \\ =& Var[R_g] \\ =& \left(\frac{M_2}{b - M_1 l} \right)^2 ((\sigma_L^2 + \sigma_B^2 - 2\rho_{LB}\sigma_L\sigma_B)l^2 + 2(\rho_{LB}\sigma_L\sigma_B - \sigma_B^2)l + \sigma_B^2). \end{aligned}$$

For the second equality, we use the fact that there is no random variable in the total cost C.

The bank's optimization problem is to find the optimal loan ratio l^* to maximize the expected utility for the accounting profit, that is,

$$\max_l E_n - \frac{1}{2} K \sigma_n^2.$$

From the first-order-condition for the optimization, we solve

$$\frac{\partial E_n}{\partial l} - \frac{1}{2} K \frac{\partial \sigma_n^2}{\partial l} = 0.$$

This leads to

$$\zeta_5 l - \zeta_6 = 0,$$

where

$$\begin{aligned} \zeta_5 =& K M_2 (b(\sigma_L^2 + \sigma_B^2 - 2\rho_{LB}\sigma_L\sigma_B) + M_1(\rho_{LB}\sigma_L\sigma_B - \sigma_B^2)) + M_1((\mu_L - \mu_B)b + \mu_B M_1) \\ & - \eta \frac{M_1}{M_2}(b(k_1 M_1 - k_2 M_2) + k_2 M_1 M_2) + \gamma\lambda b M_1, \\ \zeta_6 =& b((\mu_L - \mu_B)b + \mu_B M_1) - K M_2((\rho_{LB}\sigma_L\sigma_B - \sigma_B^2)b + \sigma_B^2 M_1) \\ & - \frac{\eta b}{M_2}(b(k_1 M_1 - k_2 M_2) + k_2 M_1 M_2) + \gamma\lambda b^2. \end{aligned}$$

Then, the optimal lending ratio l^* is

$$l^* = \frac{\zeta_6}{\zeta_5}. \tag{4.2}$$

5 Calibration Result

We have now obtained the optimal lending ratio for each model. In this section, we identify the model closest to the actual data through the calibration. We use two different data sets[1] of financial statements in the calibration. The first is to calibrate our formulae for the financial statements of five large Japanese banking companies in a fiscal year, by assuming that the estimated parameters are common among those banks (cross bank data). The second is to calibrate our formula for the historical financial statements of a large Japanese banking company (historical bank data). We assign the items of the bank's financial statement to the variables in our model as follows:

$$\begin{aligned}
L =& \text{Call loans} + \text{Loans and bills discounted},\\
B =& \text{Securities},\\
D_p =& \text{Deposits} + \text{Negotiable certificates of deposit},\\
N =& \text{Call money} + \text{Commercial papers},\\
C_r =& \text{Cash and due from banks},\\
e =& \text{Debentures} + \text{Borrowed money} + \text{Bonds payable} + \text{Bonds with share acquisition rights}\\
&+ \text{Total net assets},\\
C_0 =& \text{Interest on deposits} + \text{Interest on negotiable certificates of deposit},\\
C_N =& \text{Interest on call money} + \text{Interest on commercial papers}.
\end{aligned}$$

The parameters are estimated through the calibration as follows: for formula (3.1), we estimate μ_L, μ_B, σ_L, σ_B, ρ_{LB}, and K. With regards to formulae (4.1, 4.2), we estimate k_1, k_2, μ_L, μ_B, σ_L, σ_B, ρ_{LB}, and K. The model parameters are determined to minimize the model error, measured by the average of the squared difference between the model and actual lending ratios, that is,

$$\min_{\Pi} Error := \min_{\Pi} \sum_{\#\text{ of banks}} \frac{(l^* - l)^2}{\#\text{ of banks}}, \tag{5.1}$$

where Π is the set of parameters and l is the actual lending ratio.

5.1 Calibrated Result for Cross Bank Data

We first examine how the obtained formulae fit the actual data for five large Japanese banking companies (Mizuho, MUFG, SMBC, Resona, and Saitama Resona). Recall that all parameters to be estimated are common for all companies.

Table 1 shows the result. The accuracy of the model fitting is evaluated by "*Error*". The table shows that the P/L model achieves the minimum *Error*. The Non-P/L model is subordinated and the Simple model is the worst. Therefore, the model taking into account the profit and loss statements under the internalized balance sheet model has the best accuracy in the model fitting for the cross bank data. However, there is no significant difference between the Non-P/L and P/L models, unlike the differences between the Simple model and others.

[1] We use the data sets released by the Japanese Bankers Association.

Table 1: Estimated parameters and the accuracy of the model fit for the cross bank data. "Simple," "Non-P/L," and "P/L" correspond to the formulae (3.1), (4.1), and (4.2), respectively.

	Simple	Non-P/L	P/L
k_1	-	0.5000	0.4996
k_2	-	0.5000	0.4996
μ_L	1.1997	1.2000	1.2000
μ_B	1.0010	1.1931	1.1930
σ_L	0.0107	0.0100	0.0100
σ_B	0.0167	0.0108	0.0108
ρ_{LB}	0.0145	0.7954	0.7957
K	0.0003	0.0001	0.0001
$Error$	0.00273	0.00087	0.00083

Table 2: Estimated parameters and the accuracy of the model fitting for the historical bank data. "Simple", "Non-P/L", and "P/L" corresponds the fromula (3.1), (4.1), and (4.2), respectively.

	Simple	Non-P/L	P/L
k_1	-	0.7913	1.0000
k_2	-	0.0001	0.0001
μ_L	1.0783	1.2000	1.2000
μ_B	1.1003	1.0708	1.0000
σ_L	0.0122	0.0100	0.0100
σ_B	0.0150	0.0167	0.0143
ρ_{LB}	0.0144	0.0142	0.0159
K	0.0003	0.0003	0.0002
$Error$	0.00208	0.00018	0.00022

5.2 Calibrated Result for Historical Bank Data

Next, we investigate how our formulae fit the actual data for a banking company. Therefore, we estimate the parameters for an individual bank. As an example, we use the financial statements for FY2008-FY2018 of the MUFG.

Table 2 shows the result. The table shows that the Non-P/L model achieves the minimum *Error*, as opposed to the last examination. The P/L model is subordinated and the Simple model has the worst accuracy in the model fitting. However, there is no significant difference between the Non-P/L and P/L models, unlike the differences between the Simple model and others, as well as the last examination. Moreover, the values of *Error* for the Simple, Non-P/L, and P/L are 0.00208, 0.00018, and 0.00022, respectively. These errors are less than the values in the last examination.

5.3 Summary

We now summarize from the examinations in Section 5.1 and 5.2. The model that has all items on the liability side of the balance sheet exogenously given is the least accurate in fitting the model lending ratio to the actual lending ratio. Tables 1 and 2 show that the accuracy of the model fitness is not uniform. This means that the internalization of the liability side by the items on the asset side is more important than whether the profit and loss statements, apart from the balance sheet, is incorporated for modeling the bank's behavior in the mean-variance framework. Finally, considering the type of data set used the calibration for an individual bank is superior to the calibration for cross banks in the accuracy of the model fitting.

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