Utility Indifference Price for Asian Option in the Stochastic Volatility Model

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Abstract

This article aims to express the pricing formula of Asian option via utility indifference pricing and examine the effects of the stochastic volatility on the option price. We consider the pricing problem for Asian option under the stochastic volatility, and derive approximated utility indifference price for the option. In order to express Asian option pricing formula by probabilistic form, we use the approximation scheme for utility indifference pricing. We further explore that the effects of the skewness for the return of the underlying on Asian option price by numerical scheme as examined in Heston (1993).

1 Introduction

In this paper, we study Asian option pricing by the utility indifference pricing approach under the stochastic volatility. The utility indifference price criterion was introduced by Hodges and Neuberger (1989), and since then, it has been applied to derivative pricing problems in various incomplete market models. However, the indifference price for Asian option has never been examined yet. So, this is the first trial for it.

Many researchers have studied Asian option pricing problems. They especially interested in implementation of numerical methods in the Asian option pricing. Rogers (1995) and Věčěr (2001) developed the computational pricing method via PDE. Věčěr used dimension reduction technique, and suggested fast/accurate computational scheme. Fouque and Han (2003) considered the Asian option pricing problem under the stochastic volatility model. Since incorporating the stochastic volatility adds the variable to the option pricing problem, one might not use PDE approach. Fouque and Han utilize the dimension reduction scheme used by Věčěr's and the fast-mean reverting stochastic volatility asymptotic analysis. Approximated price derived by them takes into account implied volatility skew.

In order to derive the utility indifference price, we need to solve two distinct expected utility maximization problems. One is a problem without any claims, and the other is a problem with a claim. In various literatures, these problems have been direclty solved by HJB equation (Hodges and Neuberger (1989), Musiela and Zariphopolou (2004)). Under the restrected financial market model (Musiela and Zariphopolou (2004)), we can express the utility indifference price in explicit form. In general, PDE's however satisfied by the indifference price are non-linear in the incomplete market model such as the stochastic volatility model used in this article. We thus should rely on the numerical scheme and that has some difficulties as mentioned above, if we want to solve the indifference price. For solving these problems we apply the results of Chen *et al*., (2008), and then we obtain approximated expression of the utility indifference price. Chen *et al*., gave an approximated indifference pricing formula, and

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theoretically showed that their formulas have good approximation accuracy compared with earlier studies. We also apply their approximated method in our work.

By their approximation method, the indifference price p is expressed as

$$p = P_1 + P_2 + \cdots .$$

Terms P_i (i = 2, 3, . . .) are depended on the first-derivative of the payoff function with non-traded asset price process. In our work, the non-traded asset price corresponds the stochastic volatility level. Since we consider arithmetic Asian option, its payoff function is not function of the volatility level. The first-derivative of the payoff function with the non-traded asset price is zero. This vanishes higher-order terms, i.e., $P_i = 0$ (i = 2, 3, . . .). On the other hand, P_1 is expectation of the discounted payoff of the derivative under *minimal martingale measure* (Föllmer and Schweizer (1991)). Their scheme therefore consequently gives the pricing formula by *risk-neutral pricing method*. Fouque and Han also used this pricing formula, but they didn't discuss the derivation of this. So, our work compensates their study. We recommend to see Fouque and Han (2003), if you want to know efficient/fast computing method for Asian option pricing in the stochastic volatility model. We moreover explore that the effects of the skewness for the return of the underlying on Asian option price by numerical scheme as examined in Heston (1993).

The remaining paper is organized as follows: In the next section, we set the financial market model. We consider an arithmetic Asian option pricing in the stochastic volatility model. In Section 3, we consider the utility maximization problem and its dual formulation, and solve the dual problem by the approximation scheme. In Section 4, we derive approximated utility indifference price. In Section 5, we give computational results and show the effects of the skew from the stochastic volatility on the Asian option price.

2 Model

Let us consider the following financial market. There exists one risky asset (typically a stock) and one risk-free asset (typically a bank account). Initially, as for the bank account B, we assume that its value process is described by

$$\mathrm{d}B(s) = rB(s)\mathrm{d}s$$

for $0 \leqslant s \leqslant T$ with $B(0) = 1$.

Next, we start with setting the stock price process with the stochastic volatility. The uncertainty in this market is characterized by a probability space $(\Omega, \mathcal{F}, P)$. We then introduce a two-dimensional standard Brownian motion, and denote it by $W = (W_1, W_2)$ on $(\Omega, \mathcal{F}, P; \mathcal{F}_t)$, where $\mathcal{F}_t$ is a filtration generated by $(W(s); 0 \leqslant s \leqslant t)$ and satisfies the usual conditions. Under the above settings, the stock price S and the stochastic variance level Y (*Stochastic Volatility*) are respectively assumed to be driven by

$$\begin{aligned} \mathrm{d}S(s) &= S(s)\{\mu \mathrm{d}s + \sigma(s, Y(s))\mathrm{d}W_1(s)\}, \\ \mathrm{d}Y(s) &= a(s, Y(s))\mathrm{d}s + b(s, Y(s))\{\rho \mathrm{d}W_1(s) + \sqrt{1-\rho^2}\mathrm{d}W_2(s)\} \end{aligned} \tag{2.1}$$

where $\rho \in (-1, 1)$. The functions σ, a and b are Lipschitz continuous functions in order to ensure the existence of the unique global solution.

Next, we construct the wealth process. We assume that the the investor has wealth X at time $s \in [t, T]$, and the money amount $\pi(s)$ of X(s) is invested into the stock, the rest of the money is invested into the bank account by self-financing rule. The wealth process is then given by

$$\mathrm{d}X(s) = (rX(s) + \pi(s)(\mu - r))\mathrm{d}s + \pi(s)\sigma(s, Y(s))\mathrm{d}W_1(s). \tag{2.2}$$

We would price a European-type fixed strike Asian option with the strike price K whose payoff function is supposed to be

$$g(T) = (A(T) - K)^+$$

where $A(t) = \frac{1}{T}\int_0^t S(s)\mathrm{d}s$. From Věcěr (2001), the underlying process is give by

$$\mathrm{d}A(t) = \left(1 - \frac{t}{T}\right)\mathrm{d}S(t) =: \tau(t)\mathrm{d}S(t) = S(s)\{\mu(s)\mathrm{d}s + \hat{\sigma}(s, Y(s))\mathrm{d}W_1(s)\}$$

where $\mu(s) = \mu\tau(s)$ and $\hat{\sigma}(s, Y(s)) = \tau(s)\sigma(s, Y(s))$.

3 Dual Problem and Approximated Solution

In this section, we consider the portfolio optimization problem (particulary utility maximization problem) to derive the utility indifference price. The optimization problem is solved using the dual formula and Hamilton-Jacobi-Bellman (HJB) equation.

Let us consider the following portfolio optimization problem:

$$\left|\begin{array}{l} \text{maximize}_\pi\ E[U(X(T) - g(T))] \\ \text{subject to } \mathrm{d}X(s) = (rX(s) + \pi(s)(\mu - r))\mathrm{d}s + \pi(s)\sigma(s, Y(s))\mathrm{d}W_1(s) \end{array}\right. \tag{3.1}$$

where U is the utility function. We derive the dual formula of (3.1) according to Chen *et al*., (2008) and Rogers (2003). At first, we define Lagrange multiplier process Λ.

$$\mathrm{d}\Lambda(s) = \Lambda(s)\{\alpha(s, \omega)\mathrm{d}s + \beta_1(s, \omega)\mathrm{d}W_1(s) + \beta_2(s, \omega)\mathrm{d}W_2(s)\}$$

where α, β_1 and β_2 are adapted stochastic processes and these are determined later. At this point, by the integral by parts we have

$$\begin{aligned} \int_0^T \Lambda(s)\mathrm{d}X(s) &= \Lambda(T)X(T) - \Lambda(0)X(0) - \int_0^T X(s)\mathrm{d}\Lambda(s) - \langle \Lambda, X\rangle(T) \\ &= \Lambda(T)X(T) - \Lambda(0)X(0) \\ &\quad - \int_0^T X(s)\Lambda(s)\{\alpha(s, \omega)\mathrm{d}s + \beta_1(s, \omega)\mathrm{d}W_1(s) + \beta_2(s, \omega)\mathrm{d}W_2(s)\} \\ &\quad - \int_0^T \Lambda(s)\beta_1(s, \omega)\pi(s)\sigma(s, Y(s))\mathrm{d}s. \end{aligned} \tag{3.2}$$

On the other hand, substituting (2.2) into $\int_0^T \Lambda(s)\mathrm{d}X(s)$ gives

$$\int_0^T \Lambda(s)\mathrm{d}X(s) = \int_0^T \Lambda(s)\{(rX(s) + \pi(s)(\mu - r))\mathrm{d}s + \pi(s)\sigma(s, Y(s))\mathrm{d}W_1(s)\}. \quad (3.3)$$

Taking expectation (3.2) and (3.3) give

$$\begin{aligned} I_1 &:= E\left[\int_0^T \Lambda(s)\mathrm{d}X(s)\right] \\ &= E\left[\Lambda(T)X(T) - \Lambda(0)X(0) - \int_0^T \Lambda(s)\{\alpha(s,\omega)X(s) + \beta_1(s,\omega)\pi(s)\sigma(s, Y(s))\}\mathrm{d}s\right], \\ I_2 &:= E\left[\int_0^T \Lambda(s)\mathrm{d}X(s)\right] = E\left[\int_0^T \Lambda(s)(rX(s) + \pi(s)(\mu - r))\mathrm{d}s\right], \end{aligned}$$

respectively. Therefore, we have

$$\begin{aligned} &I_1 - I_2 \\ &= E\left[\Lambda(T)X(T) - \Lambda(0)X(0) - \int_0^T \Lambda(s)\{(\alpha(s,\omega) + r)X(s) + \beta_1(s,\omega)\pi(s)\sigma(s, Y(s)) + \pi(s)(\mu - r)\}\mathrm{d}s\right] \\ &= 0. \end{aligned} \quad (3.4)$$

By (3.4), we obtain Lagrangian L as upper bound1[1] of (3.1):

$$\begin{aligned} L(\Lambda) = \max_{X \geqslant 0, \pi} E\Bigg[&U(X(T) - g(T)) - \Lambda(T)X(T) + \Lambda(0)X(0) \\ &- \int_0^T \Lambda(s)\{(\alpha(s,\omega) + r)X(s) + \beta_1(s,\omega)\pi(s)\sigma(s, Y(s)) + \pi(s)(\mu - r)\}\mathrm{d}s\Bigg]. \end{aligned} \quad (3.5)$$

From Chen *et al.*, (2008) and Rogers (2003), the first order condition and the complementary slackness conditions to achieve the maximum value $L^*(\Lambda)$ are given as follows:

$$\frac{\partial L(\Lambda)}{\partial X(T)} = 0, \quad \frac{\partial L(\Lambda)}{\partial X(t)} = 0 \quad \text{and} \quad \frac{\partial L(\Lambda)}{\partial \pi} = 0.$$

These equations lead

$$X(T) - g(T) = I(\Lambda(T)), \quad (3.6)$$

$$\alpha(s, \omega) = -r \quad (3.7)$$

and

$$\beta_1(s,\omega) = -\frac{\mu - r}{\sigma(s, Y(s))} =: \beta_1^*(s, Y(s)), \quad (3.8)$$

respectively, where $I(\cdot)$ is the inverse function of $U'(\cdot)$. By (3.6), (3.7) and (3.8), (3.5) reduces

$$L^*(\Lambda) = E\left[\tilde{U}(\Lambda(T)) - \Lambda(T)g(T) + \Lambda(0)X(0)\right] \quad (3.9)$$

where $\tilde{U}(\cdot)$ is the convex dual function defined by

1 In (3.1), the maximized expected utility is achieved with respect to any X. On the other hand, we have the nonnegative condition of X in (3.5). This is why L in (3.5) is upper bound of (3.1).

$$\tilde{U}(y) = \sup_x [U(x) - xy]$$

and Lagrange multiplier is taken form of

$$\Lambda(T) = \Lambda(t)e^{-r(T-t)}M_1(t, T)\ M_2(t, T)$$

with

$$M_1(t,T) = \exp\left(-\int_t^T \frac{\mu - r}{\sigma(s, Y(s))} \mathrm{d}W_1(s) - \frac{1}{2}\int_t^T \left(\frac{\mu - r}{\sigma(s, Y(s))}\right)^2 \mathrm{d}s\right),$$

$$M_2(t,T) = \exp\left(-\int_t^T \beta_2(s,\omega)\mathrm{d}W_2(s) - \frac{1}{2}\int_t^T \beta_2(s,\omega)^2 \mathrm{d}s\right)$$

by (3.8). From (3.9), we obtain the dual formula

$$\min_{\Lambda} E\left[\tilde{U}(\Lambda(T)) - \Lambda(T)g(T) + \Lambda(0)\ X(0)\right]. \tag{3.10}$$

Finally, we define equivalent martingale measures.

Definition 3.1. *Define equivalent martingale measure P^* and P^{**} such that*

$$\frac{\mathrm{d}P^*}{\mathrm{d}P}\bigg|_{\mathcal{F}_t} = M_1(t,T), \quad \frac{\mathrm{d}P^{**}}{\mathrm{d}P}\bigg|_{\mathcal{F}_t} = M_1(t,T)M_2(t,T)$$

for $t \in [0, T]$.

Let us solve the dual problem. By the form of $\Lambda(T)$, we have to minimize with respect to $\Lambda(0)$ and $\{\beta_2(s, \omega)\}_{s\in[0,T]}$. We firstly consider the minimization with respect to $\Lambda(0)$. This is a static problem, so the first order condition is

$$E\left[(\tilde{U}'(\Lambda(T)) - g(T))\frac{\partial \Lambda(T)}{\partial \Lambda(0)} + X(0)\right] = 0.$$

This and Definition 1 give the optimal initial wealth

$$X^*(0) = e^{-rT}E\Big[M_1(0,T)M_2(0,T)(-\tilde{U}'(\Lambda(T)) + g(T)))\Big] = e^{-rT}E^{**}[I(\Lambda(T)) + g(T)] \tag{3.11}$$

since $\tilde{U}0(\Lambda(T)) = -I(\Lambda(\mathrm{T}))$, where E^{**} is the expectation under P^{**}. By arguments in Chen *et al.*,(2008), (3.11) is extended to

$$X^*(t) = e^{-r}(T-t)E^{**}\mathrm{t}[I(\Lambda(T)) + g(T)] \tag{3.12}$$

for $t \in [0, T]$, where the subscription t of E_t denotes condition on the information up to t.

Next, we try to solve optimization problem (3.10) with respect to $\{\beta_2(s, \omega)\}_{s\in[0,T]}$. We use Hamilton- Jacobi-Bellman equation (HJB equation), and simultaneously determine optimal $\{\beta_2(s, \omega)|s \in [0, T]\}$. We solve an optimization problem:

$$\text{minimize}_{\beta_2} \quad E[\tilde{U}(\Lambda(T)) - \Lambda(T)g(\mathrm{T})] + \Lambda(0)X(0).$$

We consider the following problem only.

$$u(t, \Lambda, A, S, Y) := \min_{\beta_2} E_t\left[\tilde{U}(\Lambda(T)) - \Lambda(T)g(T)\right].$$

At this point, we introduce a generator $\mathcal{L}$.

$$L = -r\Lambda\frac{\partial}{\partial\Lambda} + \mu(t)S\frac{\partial}{\partial A} + \mu S\frac{\partial}{\partial S} + a(t,Y)\frac{\partial}{\partial Y} + \frac{1}{2}\Lambda^2\beta_1^*(t,Y)^2\frac{\partial^2}{\partial\Lambda^2} + \frac{1}{2}\hat{\sigma}(t,Y)^2S^2\frac{\partial^2}{\partial A^2}$$
$$+ \frac{1}{2}\sigma(t,Y)^2S^2\frac{\partial^2}{\partial S^2} + \frac{1}{2}b(t,Y)^2\frac{\partial^2}{\partial Y^2} + \Lambda\beta_1^*(t,Y)\hat{\sigma}(t,Y)S\frac{\partial^2}{\partial\Lambda\partial A} - (\mu-r)\Lambda S\frac{\partial^2}{\partial\Lambda\partial S}$$
$$- \rho b(t,Y)\beta_1^*(t,Y)\Lambda\frac{\partial^2}{\partial\Lambda\partial Y} + \hat{\sigma}(t,Y)\sigma(t,Y)S^2\frac{\partial^2}{\partial A\partial S}$$
$$+ \rho\hat{\sigma}(t,Y)b(t,Y)S\frac{\partial^2}{\partial A\partial Y} + \rho b(t,Y)\sigma(t,Y)S\frac{\partial^2}{\partial S\partial Y}$$

HJB equation for the value function u is given by

$$\begin{cases} u_t + \mathcal{L}u + \min_{\beta_2}\left[\frac{1}{2}\Lambda^2\beta_2^2u_{\Lambda\Lambda} + b(t,Y)\sqrt{1-\rho^2}\Lambda\beta_2u_{\Lambda Y}\right] = 0, \\ u(T,\Lambda,A,S,Y) = \tilde{U}(\Lambda) - \Lambda g(T). \end{cases} \tag{3.13}$$

The minimum value is achieved at

$$\beta_2^*(t,\Lambda,A,S,Y) = -\frac{b(t,Y)\sqrt{1-\rho^2}u_{\Lambda Y}}{\Lambda u_{\Lambda\Lambda}}. \tag{3.14}$$

Plugging (3.14) into (3.13), we obtain the following nonlinear PDE

$$\begin{cases} u_t + Lu = \frac{1}{2}b(t,Y)^2(1-\rho^2)\frac{u_{\Lambda Y}^2}{u_{\Lambda\Lambda}} = \frac{1}{2}\Lambda^2u_{\Lambda\Lambda}(\beta_2^*)^2, \\ u(T,\Lambda,A,S,Y) = \tilde{U}(\Lambda) - \Lambda g(T). \end{cases} \tag{3.15}$$

3.1 Approximated Solution

In this section, we consider to approximate the solution of PDE (3.15). We firstly remove the nonlinear term from (3.15) by putting $\beta_2 \equiv 0$, that is, we consider the linear PDE as follows

$$\begin{cases} u_t^{(0)} + Lu^{(0)} = 0, \\ u^{(0)}(T,\Lambda,A,S,Y) = \tilde{U}(\Lambda) - \Lambda g(T). \end{cases}$$

From Feynman-Kac formula, this PDE has a solution

$$u^{(0)}(t,\Lambda^{(0)},A,S,Y) = E_t\left[\tilde{U}(\Lambda^{(0)}) - \Lambda^{(0)}g(T)\right] \tag{3.16}$$

where $\Lambda^{(0)}(t)$solves to

$$d\Lambda^{(0)}(t) = \Lambda^{(0)}(t)\{-r\,dt + \beta_1^*(t,Y(t))dW_1(t)\}.$$

We immediately obtain β_2 for the solution $u^{(0)}$ from (3.16)

$$\beta_2^{(0)}(t,\Lambda^{(0)},A,S,Y) = -\frac{b(t,Y)\sqrt{1-\rho^2}u_{\Lambda Y}^{(0)}}{\Lambda^{(0)}u_{\Lambda\Lambda}^{(0)}}. \tag{3.17}$$

Substituting $u^{(0)}$ or $\beta_2^{(0)}$ into R.H.S. of PDE (3.15), we have

$$\begin{cases} u_t^{(1)} + Lu^{(1)} = \frac{1}{2}b(t,Y)^2(1-\rho^2)\frac{(u_{\Lambda Y}^{(0)})^2}{u_{\Lambda\Lambda}^{(0)}} =: \frac{1}{2}(\Lambda^{(0)})^2u_{\Lambda\Lambda}^{(0)}(\beta_2^{(0)})^2, \\ u^{(1)}(T,\Lambda,A,S,Y) = \tilde{U}(\Lambda) - \Lambda g(T). \end{cases} \tag{3.18}$$

Feynman-Kac formula gives a solution of (3.18), that is

$$\begin{aligned} u^{(1)}(t,\Lambda,A,S,Y) &= E_t\left[\tilde{U}(\Lambda^{(0)}(T)) - \Lambda^{(0)}(T)g(T)\right] + E_t\left[\frac{1}{2}\int_t^T u^{(0)}_{\Lambda\Lambda}(s)(\Lambda^{(0)}(s))^2(\beta_2^{(0)}(s))^2\,ds\right] \\ &= u^{(0)}(t,\Lambda^{(0)},A,S,Y) + E_t\left[\frac{1}{2}\int_t^T u^{(0)}_{\Lambda\Lambda}(s)(\Lambda^{(0)}(s))^2(\beta_2^{(0)}(s))^2\,ds\right] \end{aligned} \tag{3.19}$$

where $u^{(0)}(s) := u^{(0)}(s,\Lambda^{(0)}(s),A(s),S(s),Y(s))$, $\beta_2^{(0)}(s) := \beta_2^{(0)}(s,\Lambda^{(0)}(s),A(s),S(s),Y(s))$.

4 Indifference Price

In this section, we derive the utility indifference price. At first, we give $\beta_2^{(0)}$ precisely. To this end, we calculate $u^{(0)}_{\Lambda Y}$ and $u^{(0)}_{\Lambda\Lambda}$. Note that, under $\beta_2^{(0)}$ we have

$$u^{(0)}(t,\Lambda,A,S,Y) = E_t\left[\tilde{U}(\Lambda^{(0)}(T)) - \Lambda^{(0)}g(T)\right].$$

The fact that $\frac{\partial \tilde{U}(\Lambda(T))}{\partial \Lambda(T)} = -I(\Lambda(T))$ and the linearity of expectation give the following results

$$\begin{aligned} u^{(0)}_{\Lambda\Lambda} &= -e^{-r(T-t)}E_t^*\left[I'(\Lambda^{(0)}(T))\frac{\Lambda^{(0)}(T)}{\Lambda^{(0)}(t)}\right], \\ u^{(0)}_{\Lambda Y} &= -e^{-r(T-t)}E_t^*\left[g_{Y(T)}(T)\frac{\partial Y(T)}{\partial Y(t)}\right] = 0 \end{aligned}$$

where $\Lambda^{(0)}(T) = \Lambda^{(0)}(t)e^{-r(T-t)}M_1(t,T)$. The second equation is used $g_{Y(T)} := \frac{\partial g(T)}{\partial Y(T)} = 0$. Plugging $u^{(0)}_{\Lambda\Lambda}$ and $u^{(0)}_{\Lambda Y}$ into (3.17) yields

$$\beta_2^{(0)}(t,\Lambda,A,S,Y) = 0. \tag{4.1}$$

This leads $u^{(1)}(t,\Lambda,A,S,Y) = u^{(0)}(t,\Lambda,A,S,Y)$ in (3.19).

We proceed the derivation of the indifference price. The definition of the utility indifference price $p(0)$ via duality forms is

$$\min_{\Lambda(0)} E\left[\tilde{U}(\Lambda(T)) + \Lambda(0)X(0)\right] = \min_{\Lambda(0)} E\left[\tilde{U}(\Lambda(T)) - \Lambda(T)g(T) + \Lambda(0)(X(0)+p(0))\right]. \tag{4.2}$$

Suppose that $\Lambda^{*}0(T)$ is the optimal Lagrangian level when no-claim is issued, and $\Lambda^{*p}(T)$ is the one when claims are issued, such that

$$\begin{aligned} \Lambda^{*0}(T) &= \Lambda^{*0}(t)e^{-r(T-t)}M_1(t,T), \\ \Lambda^{*p}(T) &= \Lambda^{*p}(t)e^{-r(T-t)}M_1(t,T)M_2(t,T). \end{aligned}$$

Parameter	Value
μ	0.2
ξ	2.0
$Y(0)$	0.04
$\bar{Y}$	0.04
ϕ	0.2
K	100

Table 1: Parameters

Under $\Lambda^{*0}(T)$ and $\Lambda^{*p}(T)$, L.H.S. of (4.2) reduces

$$E\left[\tilde{U}(\Lambda^{*0}(T))\right] + \Lambda^{*0}(0)X(0), \tag{4.3}$$

and R.H.S. of (4.2) is

$$\begin{aligned} &E\left[\tilde{U}\ (\Lambda^{*p}(T)) - \Lambda^{*p}(T)g(T)\right] + \Lambda^{*p}(0)(X(0) + p(0)) \\ &\approx \mathrm{u}^{(1)}(0,\Lambda^{*0}, A, S, Y) + \Lambda^{*p}(0)(X(0) + p(0)) \\ &= \mathrm{u}^{(0)}(0,\Lambda^{*0}, A, S, Y) + \Lambda^{*p}(0)(X(0) + p(0)). \end{aligned} \tag{4.4}$$

From (4.1), it holds $M_2(t, T) = 1$ and this leads

$$\Lambda^{*p} = \Lambda^{*p}(t)e^{-r(T-t)}M_1(t, T) = \Lambda^{*0}. \tag{4.5}$$

Note that, (4.5) is also shown in Chen *et al*., (2008). They derived it by Taylor expansion. From (4.3), (4.4) and (4.5) with the definition of the indifference price, we have the approximated indifference price

$$p(0) = e^{-rT}E^{*}\ [g(T)]\ . \tag{4.6}$$

This result is just a pricing formula under *minimal martingale measure*.

5 Numerical Result

In this section, we explore the effects of the skewness for the return of the underlying on Asian option price as examined in Heston (1993). We use Heston model as example of the stochastic volatility model. That is, we take $a(s, Y(s)) = \xi(\overline{Y} - Y(s))$ and $b(s, Y(s)) = \phi\sqrt{Y(s)}$ in (2.1), where ξ, $\overline{Y}$ and ϕ are constant. The parameters used in this examination are shown in Table 1.

Figure 1 shows simulated density curves of the spot return on the underlying $\{A(s)\}_{s\in[0,T]}$ for each ρ ($\rho = -0.5, 0.5$). From Figure 1, we observe the skewness for $\rho = -0.5, 0.5$ compared with Black-Scholes price process. The density curve at $\rho = -0.5$ (dotted line) has a fat left tail and a thin right tail. The density curve at $\rho = 0.5$ (dashed line) has a thin left tail and a fat right tail.

The skewness also affects Asian option price. Figure 2 shows it. This describes that option price differences between the stochastic volatility model and Black-Scholes model (call BS *price*). In ITM area, the price at $\rho = -0.5$ (dotted line) is larger than BS price. On the other hand, in OTM area, the price at $\rho = 0.5$ (dashed line) is larger than BS price. These biases are corresponding to the skewness described in Figure 1.

These results have same characteristics to plane vanilla option in Heston's results.

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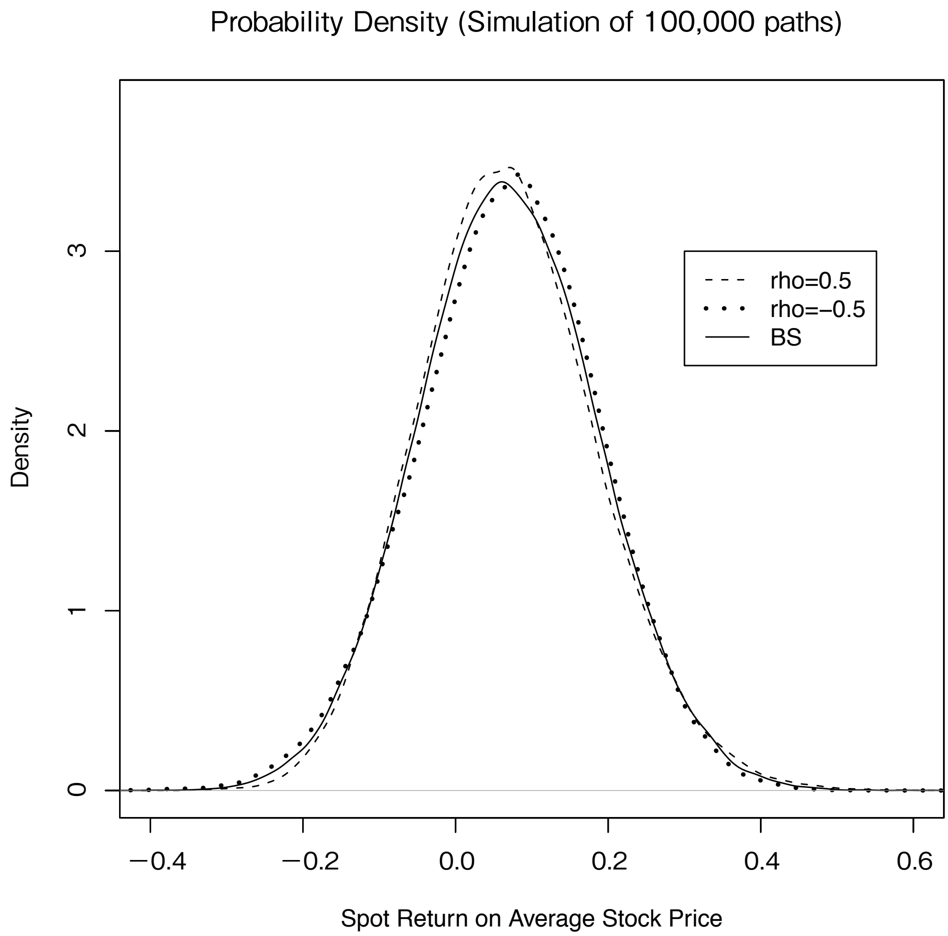

Figure 1: Comparison of probability density functions for the spot returns on the average stock prices A. These functions are described by the numerical simulation with 100,000 paths.

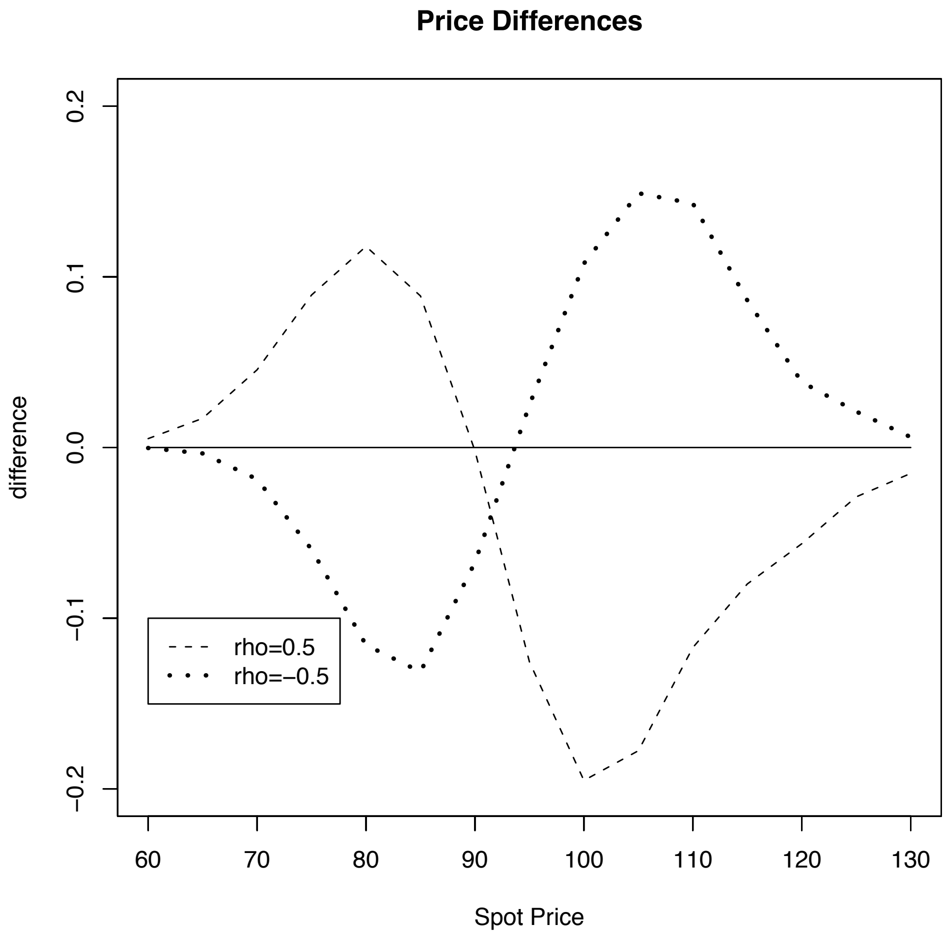

Figure 2: Differences of the Asian option price between Black-Scholes model and the stochastic volatility model. The dot-line shows the option price difference between Black-Scholes model and the stochastic volatility model with $\rho = -0.5$, and vice versa.