A Study of pairwise-comparison interfaces in AHP

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Abstract

Pairwise comparison is the most important element to be used when analysis in the AHP. Pairwise comparison is a method to calculate the weight for each element perform a comparison of two advantages. The consistency of an answer will be determined by the value of C. I. in a pairwise comparison table. If its value is to be considered for the case exceeds 0.1 (Saaty's criteria 1980)[5] is not consistent, the pairwise comparison is reiterated. It is a challenge to resolve a long-standing weakness of AHP, is generated by the research cost together and again it is not easy. This article aims to consider the selection of an efficient interface when performing a pairwise comparison to reduce the costs associated with AHP.

Keywords: analytic hierarchy process, consistency index, pairwise comparison, interface

1. Introduction

The Analytical Hierarchy Process (AHP) is a multi-objective decision-making method developed by Saaty[5−8]. It aims at quantifying relative priorities for a given set of alternatives on a ratio scale, based on the judgment of the decision-maker, and stresses the importance of the intuitive judgments of a decision-maker as well as the consistency of the comparison of alternatives in the decision-making process[5]. Since a decision-maker bases a judgment on knowledge and experience, then makes decisions accordingly, the AHP approach matches the behavior of a decision-maker. The strength of this approach is that it organizes tangible and intangible factors in a systematic way, and provides a structured, yet relatively simple solution to the decision-making problems[11]. Moreover, The Consistency Index (C. I.) is used to measure the reliability of the pairwise-comparison of AHP. A pairwise-comparison should be carried out carefully because all factors shuld be included, such as time cost. Satty consistency index C. I. is an AHP method which applies typical criteria to check the reliability of the pairwise comparison matrix value. Regards to the usefulness of C. I., examined a number of numerical experiments, have been made ((Shibayama Nishina1992, etc.), (Satty2003)[11].

The objective of this paper to evaluate an efficient interface of pairwise comparing in AHP. The paper will briefly review the concepts and meaning of C. I. in the AHP; the AHP's implementation steps, and demonstrate the various interfaces of pairwise comparing in AHP. It is hoped that this will encourage its application in a wide range of decision-making problems.

2. AHP & CI

Analytic Hierarchy Process (AHP), since its invention, has been a tool available to decision-makers and researchers and is one of the most widely used multiple criteria decision-making tools (Vaidya and Kumar 2006)[4]. It is designed to cope with both the rational and the intuitive to select the best from a number of alternatives evaluated with respect to several criteria. In this process, the decision maker carries out simple pairwise comparison judgments, which are then used to develop overall priorities for ranking the alternatives (Saaty and Vargas 2001)[9].
The form of matrix of the pair-wise comparisons is as follows:

$$A=\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array}\begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \end{array} \\ \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_1/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix} \end{array}$$

The comparisons are made using a scale that indicates the importance of one element over another element with respect to a given attribute. Table 1 shows the scale ranges from 1 for 'least valued than' to 9 for definitely most important than.

Table 1. 1–9 Scale for the pair-wise comparison (Saaty 2001)

Linguistic term	Preference number
Equally important	1
Slightly more important	3
more important	5
much more important	7
most important	9
Intermediate values	2, 4, 6, 8

In the basic structure of an Analytic Hierarchy presented in Figure 1, the goal is specified at the top, all the objectives or criteria are listed below the goal and all alternatives are presented at the last level.

Some key and basic steps involved in this methodology are;

Step 1. Determine the problem.

Step 2. Structure the decision hierarchy of different levels constituting goal, criteria, sub-criteria and alternatives.

Step 3. Compare each element in the related level and establish priorities.

Step 4. Perform calculations to find the normalized values for each criteria/alternative. Calculate the maximum Eigen value and C. I..

Step 5. If the maximum Eigen value, C. I. is satisfactory, then the decision is made based on the normalized values. If not, the procedure is repeated until the values lie in the desired range.

The consistency analysis is a part of the AHP method. It is applied in order to assure a certain

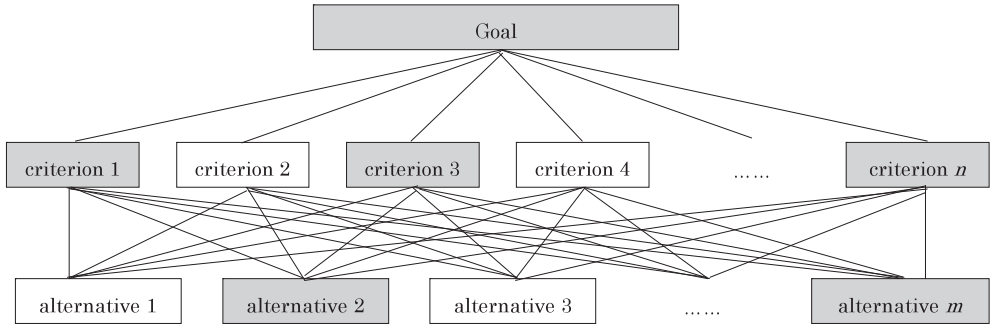

Figure 1. Basic structure of AHP

quality level of decision. The measure of inconsistency can be used to successively improve the consistency of judgments (Saaty and Vargas 2001)[9]. The formula 2 and 3 is generated to determine the convenience of the numerical judgment. In this respect, we calculated the C. I. confirming Saaty, which is defined as a ratio between the consistency of a given evaluation matrix and the consistency of a random matrix.

3. Consistency analysis

Once the model is built, the decision-makers evaluate the elements by making pairwise comparisons. When all the comparisons are completed, we calculate the priorities and a measure of consistency of our judgment. It is applied in order to assure a certain quality level of decision. The measure of inconsistency can be used to successively improve the consistency of decision-making (Saaty 2001)[9]. The equation below is generated to determine the convenience of the pairwise-comparison. The C. I. is not less than 0.10. In this respect, we calculated the C. I. confirming of pairwise comparing to improve the reliability of AHP. C. I. is related to the eigenvalue method:

$$CI = \frac{\lambda_{\max} - n}{n - 1},$$

where $\lambda_{\max}$ indicates maximal eigenvalue.

But, the C. I. is very sensitive to the task of pairwise-comparison by the decision maker. It takes significant costs (e.g. time, money) when the decision maker fills a pairwise-comparison matrix. A pairwise comparison is the process of comparing the relative importance, preference, or likelihood of two elements with respect to an element in the level above. A comparison is made with respect to each pair (the number of comparisons will be $|u(u-1)/2|$, where u is the number of criteria in the model).

One of AHP's strengths is the possibility to evaluate quantitative as well as qualitative criteria and alternatives on the same preference scale of nine levels. These can be a numerical, verbal or graphical scale as below figure 2. (Ishizaka Alessio and Labib Ashraf 2009)[2].

Moreover, traditional text input interface had been used as in figure 3 below:

In a text input interface, the decision maker inputs the values directly, but the graphic interface is

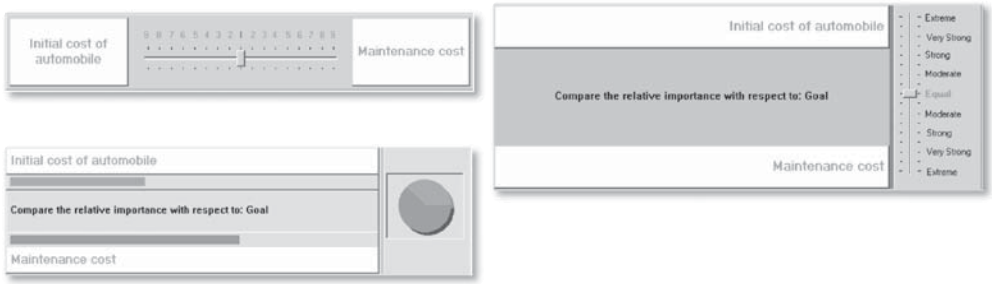

Figure 2. Various graphical interfaces of the judgment scale

criterion	A	B	C	D
A	1	3	1/5	7
B		1	3	5
C			1	3
D				1

Figure 3. Text input interfaces of the judgment scale

more intuitive due to the visual input method for the ratio. We found that the interface has a strong influence on the value of C. I. Moreover, the graphical interface has been widely used because of its convenience.

4. Experiment on the interface based on C. I.

It considered that the value of C. I. is very sensitive to the interface in a pairwise-comparison. In this paper, we compare two interfaces to identify their influence and effectiveness. In our experiment, we tested 50 people using two interfaces and then compared the C. I. values A text input interface and graphic interface, were used, We compared the values of C. I. to evaluate their effectiveness. We prepared two AHP programs developed by RealStudio on an MacOS platform as pictured below.

Figure 4. Text input interface

Figure 5. Graphic input interface

Figure 4 and figure 5 show two different interfaces, the text input type and the graphic input type. Figure 4 shows the traditional text input interface that inputs a point to the cell directly, and figure 5 is a graphic input interface that inputs a point with a level bar or button. We tested these interfaces with about 50 people. Test results are summarized in Table 2 as below. In the table, TEXT C. I. indicates the value of C. I. from the Figure 4. GRAPHIC C. I. is the value of C. I. using the interface pictured in figure 5.

From the experimental results, we know that the TEXT C. I. is less than the GRAPHIC C. I. It means that the graphic interface is not always as intuitive as the text interface in AHP. For a ratio scale problem like AHP, It is valid to input a number directly.

5. Concluding Remarks

In this paper we investigated two interfaces to reduce the value of the C. I., which occurs when

Table 2. Experimental results (the value of C. I.)

No	TEXT C. I.	GRAPHIC C. I.
1	0.06	0.02
2	0.12	0.03
3	0.2	0.09
4	0.08	0.08
5	0.06	0.09
6	0.07	0.21
7	0.06	0.32
8	0.04	0.04
9	0.07	0.08
10	0.09	0.09
11	0.05	0.09
12	0.02	0.05
13	0.4	0.24
14	0.08	0.07
15	0.04	0.08
16	0.02	0.05
17	0.05	0.09
18	0.12	0.15
19	0.32	0.07
20	0.02	0.07
21	0.06	0.04
22	0.07	0.08
23	0.06	0.33
24	0.08	0.09
25	0.08	0.02
26	0.04	0.14
27	0.12	0.03
28	0.37	0.34
29	0.05	0.08
30	0.04	0.13
31	0.08	0.05
32	0.06	0.22
33	0.09	0.09
34	0.08	0.13
35	0.03	0.04
36	0.2	0.09
37	0.12	0.09
38	0.06	0.08
39	0.04	0.28
40	0.04	0.08
41	0.07	0.15
42	0.06	0.09
43	0.45	0.07
44	0.03	0.08
45	0.07	0.33
46	0.09	0.21
47	0.04	0.08
48	0.23	0.12
49	0.03	0.08
50	0.08	0.23
MEAN	0.0978	0.1176

are over than 0.1

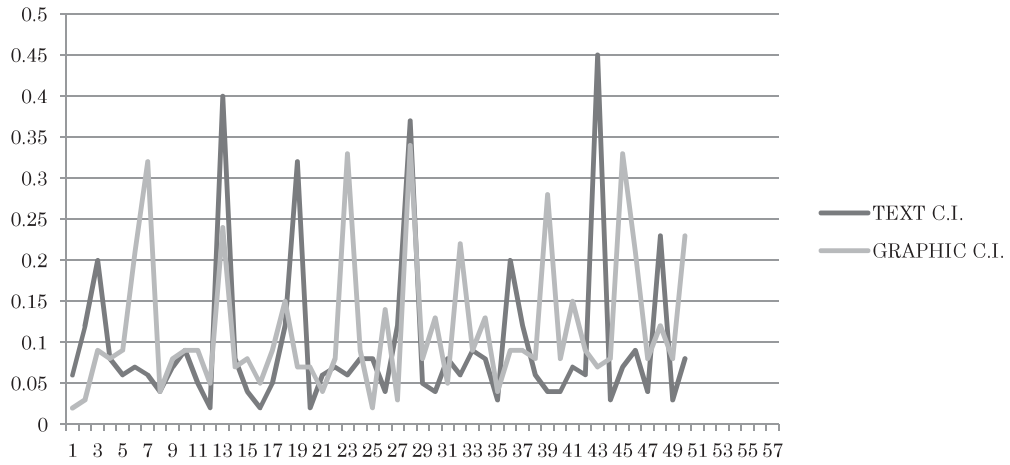

Figure 6. A graph of both C. I.

pairwise comparison is used as input in decision-making problems. The pairwise comparison matrices are widely considered as successful means for gathering data in many real world decision-making situations. The AHP and some of its variants have found a considerable appeal in approaching many real life applications. Many computer packages have been developed to automate their applications to solve real life problems.

In general, the graphic interface is used more widely than the text input interface. Nevertheless, we could found that a text input interface is more effective than a graphic interface.

Many people preferred graphic interfaces over text input interface because they believe that graphic interfaces, are more intuitive than text input interfaces. Nevertheless, this experiment shows that a text input interface is more useful to a proportional scale problem like AHP. We hope that this research can contribute to reducing the cost of AHP interview research.

Our futuer research will develop AHP support system using a Meta-heuristic method such as a genetic algorithm.

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