A study on Controlling Consistency Index of AHP
—Comparing evolutionary algorithm with random search algorithm—

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Abstract: Consistency Index is the most important measure for the results from pairwise comparison in the AHP. Pairwise comparison is a method to calculate the weights for each element that performs a comparison of two advantages. The consistency of the answer will be determined by the value of C.I. of pairwise comparison table, its value is to be considered for the case exceeded 0.1 (Satty's criteria 1980). However, when a pairwaise comparison is performed again, there is a case of not being consistent. The method of answering to the question is also a long-standing weakness of AHP. It is necessary for AHP to need a many research cost and to perform a pairwise comparison again in some cases. This article focuses to develop the efficient method that decreases the value of C.I. using evolutionary algorithm like genetic algorithm and Random Search Algorithm. Finally, we propose software and discuss its effectiveness.

Keywords: *analytic hierarchy process, consistency index, genetic algorithm, random search algorithm*

1. Introduction

The Analytical Hierarchy Process (AHP) is a multi-objective decision-making method developed by Saaty [5-8]. It aims at quantifying relative priorities for a given set of alternatives on a ratio scale, based on the judgment of the decision-maker, and stresses the importance of the intuitive judgments of a decision-maker as well as the consistency of the comparison of alternatives in the decision-making process [5]. Since a decision-maker makes judgments, a decision based on knowledge and experience, the AHP approach matches the behavior of a decision-maker. The strength of this approach is that it organizes tangible and intangible factors in a systematic way, and provides a structured yet relatively simple solution to the decision-making problems [11]. Moreover, The Consistency Index (C.I.) is used to measure the reliability of the pairwise-comparison of AHP. A pairwise-comparison should be carried out carefully because all factors should be included, such as time cost. Satty consistency index C.I. is an AHP method, which applies typical criteria to check the reliability of the pairwise comparison matrix value. Shibayama, Nishina (1992) [12] and Satty (2003) [11], have examined a number of numerical experiments regarding to the usefulness of C.I..

The objective of this paper is to propose an efficient algorithm for C.I. in AHP by comparing Genetic Algorithm (GA) with Random Search Algorithm (RSA). The paper will briefly review the concepts and meaning of C.I. in the AHP; the AHP's implementation steps, and demonstrates the various interface of pairwise comparison in AHP. It hopes that this will encourage its application in a wide range

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of decision-making problems.

2. AHP & CI

Since its invention, Analytic Hierarchy Process (AHP) has been a tool available to decision-makers and researchers and is one of the most widely used multiple criteria decision-making tools (Vaidya and Kumar 2006) [4]. It is designed to cope with both the rational and the intuitive decision maker's to select the best from a number of alternatives evaluated with respect to several criteria. In this process, the decision maker carries out simple pairwise comparison judgments, which are then used to develop overall priorities for ranking the alternatives (Saaty and Vargas 2001) [9].

The form of matrix of the pair-wise comparisons is as follows:

$$A = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \end{array} \\ \begin{pmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_1/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{pmatrix} \end{array}$$

The comparisons are made using a scale that indicates the importance of one element over another element with respect to a given attribute. Table 1 shows the scale ranges from 1 for 'the least valued than' to 9 for 'the most important than'.

Table 1. 1-9 Scale for the pair wise comparison (Saaty 2001)

Linguistic term	Preference number
Equally important	1
Weakly more important	3
Strongly more important	5
Very strong important	7
Absolutely more important	9
Intermediate values	2, 4, 6, 8

In the basic structure of an Analytic Hierarchy presented in Figure 1, the goal is specified at the top, all the objectives or criteria are listed below the goal and all alternatives are presented at the last level.

Some key and basic steps involved in this methodology are;

Step 1. Determine the problem.

Step 2. Structure the decision hierarchy of different levels constituting goal, criteria, sub-criteria and alternatives.

Step 3. Compare each element at the related level and establish priorities.

Step 4. Perform calculations to find the normalized values for each criteria / alternative. Calculate the maximum Eigen value and C.I..

Step 5. If the maximum Eigen value, C.I. is satisfactory, then the decision is made based on the normalized values. If not, the procedure is repeated until the values lie in the desired range.

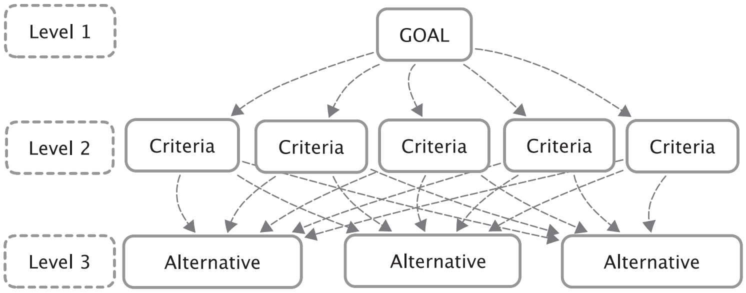

Figure 1. Basic structure of AHP

The consistency analysis is a part of the AHP method. It is to assure a certain quality level of the decision. The measure of inconsistency can be used to successively improve the consistency of judgments (Saaty and Vargas 2001) [9]. The formula 2 and 3 is generated to determine the convenience of the numerical judgment. In this respect, we calculated the C.I. confirming Saaty, which is defined as a ratio between the consistency of a given evaluation matrix and the consistency of a random matrix.

3. Consistency analysis

Once the model is built, the decision-makers evaluate the elements by making pairwise comparisons. When all the comparisons are completed, we calculate the priorities and the measure of consistency in our judgment. This is to assure the quality level of decision. The measure of inconsistency can be used to successively improve the consistency of decision-making (Saaty 2001) [9]. The equation below is generated to determine the convenience of the pairwise-comparison. The C.I. is not less than 0.10. In this respect, we calculated the C.I. confirming of pairwise comparing to improve the reliability of AHP. C.I. is related to the eigenvalue method:

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n - 1},$$

where $\lambda_{\max}$ indicates maximum eigenvalue.

But, the C.I. is very sensitive to the task of pairwise-comparison by the decision maker. It costs significantly costs (time, money) when the decision maker fills a pairwise-comparison matrix. A pairwise comparison is the process of comparing the relative importance, preference, or likelihood of two elements with respect to an element in the level above. A comparison is made with respect to each pair (the number of comparisons will be $\{u(u-1)/2\}$, where u is the number of criteria in the model).

One of AHP's strengths is the possibility to evaluate a decision maker's mental importance quantitatively.

Typically pairwise comparison is based on the professionals' experience and knowledge, but the inconsistency may happen, when dealing with many comparative elements. Although the most advanced instruments are proposed, it is difficult to obtain consistency in practice, which makes it necessary to have a method capable of evaluating the importance of this precision to a specific problem. In this paper, we consider that inconsistency is a violation of proportionality, which can sometimes mean the violation of transitivity.

4. Experimental Method for Genetic Algorithm and Random Search Algorithm

It is believed that the form of responses is necessary to identify the property of the answer. In this research, we deal with a Random Search Algorithm (RSA) in this experiment as a modification to compare it with GA.

The Genetic Algorithms (Holland, 1975) derive from the nature's evolution model. They are probabilistic research computational models that copy the natural evolution of the species, combining adaptation and survival concepts of the most capable individuals. They consist in groups of individuals and genetic operators that influence this population. They are computationally simple and powerful when it comes to optimal solutions (Eiben and Smith, 2003). Some of the articles that use AHP and GA combined successfully are Ichimura et al. (1994), Yeh et al. (1999), Moneim (2008), Abessi et al. (2003) and Fujino et al. (2009).

In our experiment, we tested both GA and RSA approaches to certify the effectiveness of decreasing the value of C.I.. Presented GA, moreover AHP programs are developed by Visual Studio 10 on Windows 7 platform and summarized with Excel.

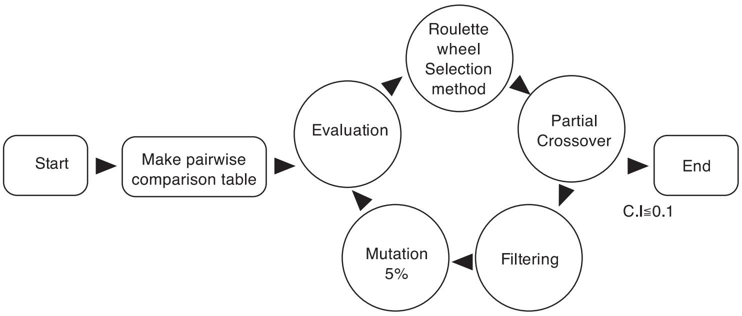

Figure 2. Genetic Algorithm Process for AHP

Figure 2 shows the algorithm that consisted with GA and the weight filtering system. Filtering system is a system to control the Crossover in GA to prevent destroying the original direction of the decision-makers.

A random search algorithm is some kind of randomness or probability (typically in the form of a pseudo-random number generator) in the definition of the method, and in the literature, called a Monte Carlo method or a stochastic algorithm. Random search methods have been shown to have a potential to solve complex problems efficiently in a way that is not possible for deterministic algorithms. Whereas it is known that a deterministic method for global optimization is NP-hard problem. Figure 3 shows the algorithm that consisted of RSA.[14]

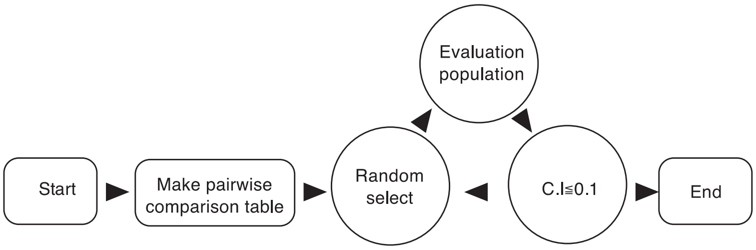

Figure 3. Random Search Algorithm for AHP

5. Experiment & Results

We performed numerical experiments on AHP pairwise-comparison sample data and the results are shown in from Table 2 to Table 5. The experimental Software that used in this research is developed by Visual Studio on Windows platform.

Table 2. Sample raw Data (C.I. = 1. 229)

	A	B	C	D	E	Weight
A	1	1	1/2	1/4	1/9	0. 075
B	1	1	1/6	8	8	0. 282
C	2	6	1	1/6	8	0. 306
D	4	1/8	6	1	1	0. 219
E	9	1/8	1/8	1	1	0. 119

Table 3. Pairwise Comparison Table after experiment

Genetic Algorithm (1)

	A	B	C	D	E	F	G	H	I	Weight
A	1	1/2	1/3	1/4	1/3					0. 073
B	2	1	1	1	4					0. 261
C	3	1	1	1	6					0. 307
D	4	1	1	1	2					0. 261
E	3	1/4	1/6	1/2	1					0. 099
F										
G										
H										
I										

Random Search (1)

	A	B	C	D	E	F	G	H	I	Weight
A	1	1/3	1/5	1/3	1/3					0. 062
B	3	1	1/2	5	4					0. 326
C	5	2	1	2	2					0. 346
D	3	1/5	1/2	1	1					0. 130
E	3	1/4	1/2	1	1					0. 136
F										
G										
H										
I										

Genetic Algorithm (2)

	A	B	C	D	E	F	G	H	I	Weight
A	1	1/4	1/4	1/4	1/7					0. 050
B	4	1	1	2	2					0. 293
C	4	1	1	1	4					0. 293
D	4	1/2	1	1	2					0. 222
E	7	1/2	1/4	1/2	1					0. 143
F										
G										
H										
I										

Random Search (2)

	A	B	C	D	E	F	G	H	I	Weight
A	1	1/4	1/5	1/3	1/3					0. 058
B	4	1	1	3	5					0. 374
C	5	1	1	1/2	3					0. 247
D	3	1/3	2	1	2					0. 217
E	3	1/5	1/3	1/2	1					0. 104
F										
G										
H										
I										

Genetic Algorithm (3)

	A	B	C	D	E	F	G	H	I	Weight
A	1	1/3	1/4	1/3	1/2					0. 073
B	3	1	1/2	4	3					0. 306
C	4	2	1	1	3					0. 324
D	3	1/4	1	1	2					0. 186
E	2	1/3	1/3	1/2	1					0. 111
F										
G										
H										
I										

Random Search (3)

	A	B	C	D	E	F	G	H	I	Weight
A	1	1/4	1/4	1/2	1/5					0. 060
B	4	1	1	6	2					0. 357
C	4	1	1	2	3					0. 310
D	2	1/6	1/2	1	1					0. 115
E	5	1/2	1/3	1	1					0. 159
F										
G										
H										
I										

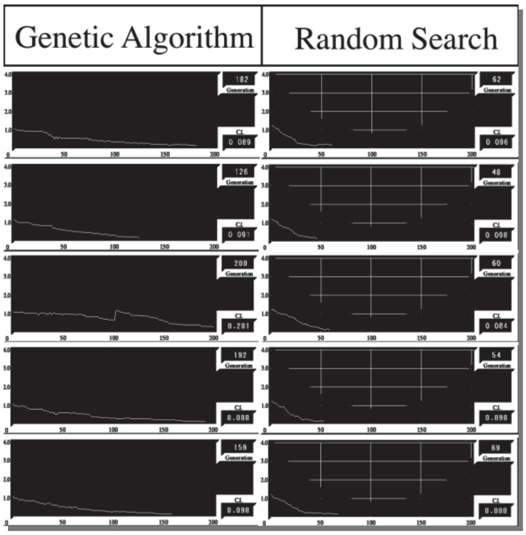

Figure 4. Process of GA and RSA (the value of fitness)

Table 3 and Figure 4 show the pairwise comparison value that represents the value of experiment. We know that the result of algorithm is very effective from the table 3.

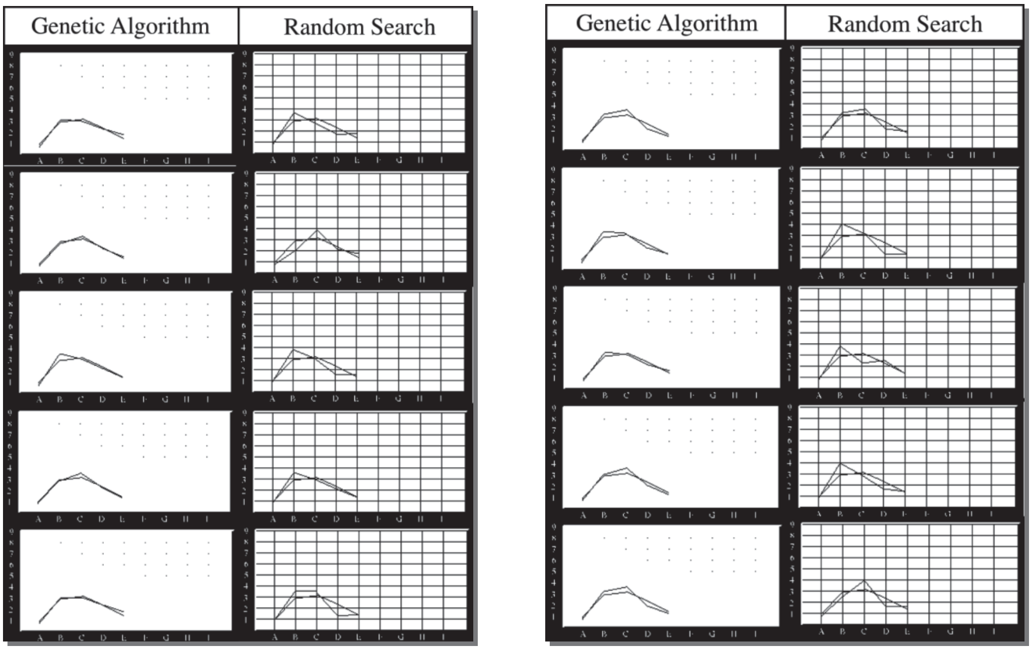

Figure 5 . Compare GA with RSA

Figure 5 shows displacement of the weight in the time of correction according to a questionnaire completed. Left-hand side is the weight corrected by GA, and right-hand side is the weight corrected by RSA. GA is considered for a gap to be comparatively smaller than RSA from the weight before correction. It is desirable that correcting C.I. is along with a decision maker's intention. By this experiment, we know that GA is suitable for correction of C.I. on AHP.

6 . Concluding and Discussion

The main objective of this paper is to offer a possible solution to inconsistency in pairwise comparison matrices using Genetic Algorithms and Random Search Algorithm. Moreover, we compared GA with RSA to (identify which is more) effective method. It is very important that inconsistency in decision analysis must be treated in order to minimize the losses and make the decision process easier. In our approach, we can find the superiority of the modification by GA than RSA. In particular, RSA is better performance than GA, but RSA did not maintain the priority thinking of the decision-maker.

The importance of the Genetic Algorithms in this research relates to the many advantages that this kind of computational method offers, among which the possibility of generating more than one solution to each pairwise comparison matrix with an inconsistency problem, allowing the decision maker to have a greater number of options to replace the original pairwise comparison matrix. In GA, we adopted a filtering system that is a system to control the Crossover process to prevent destroying the original direction of the decision makers. [Han 2013]

There was a good performance of our approach, requiring no user intervention. Although these situations are very common in Genetic Algorithms, the system is still useful. In this paper, it was relatively short, considering the dimensions of the proposed problem, quite close to the expected time, which is an advantage of the model.

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