Regime switching term structure model
—An application to Japanese corporate bond yield spreads—

Takeshi Kobayashi

Abstract

The purpose of this study is to develop a regime-switching extension of the dynamic Nelson-Siegel and its estimation methodology and apply it to Japanese corporate bond spreads data on individual firm basis. Term structure model with regime shifts has superior in-sample fit than the model without regime shifts. The results indicate estimated regime probability is closely linked to business and market sentiment. This study makes a contribution to application of the fixed income financial product such as emerging sovereign bond yield or Credit Default Swap which has nonlinear properties of time series under sovereign debt crisis.

Keywords: Term Structure Model; Regime Shifts; Credit Spreads; Macro Variable

1 Introduction

Recent study on the term structure of credit spreads explains the dynamism of credit spreads using affine term structure model or Nelson-Siegel [1987] model. These models are consistent with the result of principal component analysis in which the first -second -third factors explain over 90% of variation of credit spreads.

Nevertheless, the question has remained unanswered whether the current term structure model explains 100% of the variation of the yield curve considering non-normality or discontinuity of credit spreads.

In the research regarding the term structure of government bond yield, the models incorporating jump process or regime shifts have been developed taking into consideration the non-normality or discontinuity of government bond yield. The development of term structure with regime shifts was first introduced by Hamilton [1989], Garcia and Perron [1996], Gray [1996]. They developed and estimated time series models to capture the dynamism of short term interest rate.

Landen [2000] solved the close form solution of bond price with two regime shifts using Gaussian model. Bansal and Zou [2002] studied two factor CIR model which parameters shifts not only mean reversion but also volatility and market price of risk. Both researches insist that regime switching model has a better goodness-of-fit than the three factor affine term structure model and that regime is closely linked with business cycle and monetary policy.

However, as Litterman and Schenkman [1991] point out, over 90% variation of the yield curve is explained by three principal components (level, slope and curvature) of the term structure. One factor or two factor model need to be modified even if they admit regime shifts.

To overcome this difficulties Dai and Singleton [2003] developed multi-factor affine term structure model with regime shift. Dai, Singleton and Yang [2007] developed discrete time three factor regime switching term structure model. Their model is composed of three factors and deal with market price of risk with regime- shifts. They also show the term structure of historical volatility by each regime which is suggestive for practitioner as it is applicable for risk management and investment strategy.

There are number of studies such as Bernadell, Coche and Nyholm [2005], Nyholm [2007] and Zhu and Rahman [2009] who develop Nelson-Sigel model with regime shifts for government bond. Bernadell, Coche and Nyholm [2005] and Nyholm [2007] estimate the model in which the slope factor shifts into three regimes and investigate the relation between regime and business cycle. Zhu and Rahman [2009] present and estimate a regime switching macro-finance model of the term structure with latent and macroeconomic factors. The joint dynamics of the yield and macro factors are examined simultaneously. They point out two regimes do not fully explain the business cycle.

Little study has been done to the regime switching term structure model regarding credit spreads with the exception of Dionne, Gauthier, Hammami, Maurice and Simonato [2011][1].

They examine the ability of observed macroeconomic factors and the possibility of changes in regime to explain the proportion of yield spreads caused by the risk of default in the context of a reduced form model. They find that our macroeconomic factors are linked with two out of three sharp increases in the spreads during this sample period, indicating that the spread variations can be related to macroeconomic undiversifiable risk.

In summary of the previous research the regime switching term structure mode is the new field of research which began in early 2000. The type of model contains Nelson Siegel models and affine term structure models. They extend the regime shifts in the term structure of mean reversion, volatility and market price of risk. However little research has been made for credit spreads.

The purpose of this study is to develop a regime-switching extension of the dynamic Nelson-Siegel and its estimation methodology and apply it to Japanese corporate bond spreads data on individual firm basis. The term structure model with regime shifts might be applicable to the fixed income financial products such as emerging market bond or Credit Default Swap which has nonlinear properties of time series under sovereign debt crisis.

The remainder of this paper is structured as follows. In section 2, data and regime switching term structure model are described. In section 3, estimation results are demonstrated and section 4 concludes with implications of study results for future credit spreads modeling.

2 Data and modeling

2.1 Estimation for data

2.1.1 Government bond yield

End-of-month price quotes for Japanese Government bonds from April 1997 through December

[1] Maalaoud, Dionne and Francois [2009] and Alexander, C. and A. Kaeck [2008] propose regime switching model of credit spreads and Credit Default Swap with single maturity.

2011 were used, taken from Japan Bond Trading Co., Ltd. from maturity 1 year and 20 year. Because not every month has the same maturities available, I linearly interpolate nearby maturities to pool into fixed maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108, and 120 months. Government bond yield curve data are constructed using the Fisher, Nychka and Zervos [1995] approach.

2.1.2 Corporate bond yield

OTC Bond Transactions in the Japan Securities Dealers Association are used. Our sample comprises industrial, banking, and services sector firms in the Japan Securities Dealers Association at any time during the period Apr1997-Dec2011. It is necessary to have price data covering short to long time to maturity of corporate bond for a lot of companies at each time period. The criteria for selecting corporate bonds is constructed in the following manner.

1. Observation period: the firms whose time series has over 6 years during the period Apr1997-Dec2011 are selected. Data period starts at April 1997 in that the Japan Securities Dealers Association publish data since April 1997.
2. Time to maturity: We require corporate bonds of different maturities that have at least 7 years for each month to estimate the level, slope and curvature factors of credit spreads.
3. Number of prices: We require a minimum of 5 prices of bonds of different maturities.

Based on the above rule our final sample comprises 56 firms. Corporate bond spread is estimated by B spline model of Steely [1991] in this chapter[2]. Corporate bond spread is created in a way that corporate bond yield is subtracted from the same maturity of government bond yield.

2.2 State-Space representation of the model

To estimate this model, I introduce a unified state-space modeling approach that lets us simultaneously fit the credit curve at each point in time and estimate the underlying dynamics of the factors. This section explains the state-space representation of term structure model and estimation methodology.

2.3 Nelson-Siegel model with regime shifts

This chapter deals with Dynamic Nelson-Siegel which term structure parameter is time-varying as a base model proposed by Diebold and Li [2006][3]. First, I explain the dynamic Nelson-Siegel and then develop the Nelson-Siegel model with regime shifts. The term structure factor of Nelson-Siegel model $\beta_j(t) = (\beta_{j1}(t), \beta_{j2}(t), \beta_{j3}(t))'$ is regarded as the state variable in state-space modeling.

---

[2] B spline model is used for constructing zero coupon yield of corporate bond instead of bootstrap method as B spline model fits better than bootstrap methods for the whole sample during April 1997 to Dec 2011.

[3] Nelson-Siegel model has two type of models. One is the general case in which state variables are correlated. The other is the special case in which states variables are independent. The latter is dealt with in this study, firstly because it performs better than the former in terms of in-sample fitting and out of sample predictive power, and secondly because the number of unknown parameter is less than that of the general case.

for $j\,(j=1, ..., 56)$. The following is the definition.

- T: length of the time series.
- $\beta(t) = (\beta_1(t), \beta_2(t), \beta_3(t))' \in \mathcal{R}^3$: the term structure factor at the time of t.
- $\mu = (\mu_{\beta 1}, \mu_{\beta 2}, \mu_{\beta 3})' \in \mathcal{R}^3$: mean reversion parameter of term structure factor $\beta(t)$.
- $\boldsymbol{A} = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \in \mathcal{R}^{3\times 3}$: matrix of coeffcient of autoregressive process.
- $\eta \in \mathcal{R}^3$:disturbance term.
- $\boldsymbol{Q} = \begin{pmatrix} q_{11}^2 & 0 & 0 \\ 0 & q_{22}^2 & 0 \\ 0 & 0 & q_{33}^2 \end{pmatrix} \in \mathcal{R}^{3\times 3}$: conditional covariance matrix of term structure factor.
- $\varepsilon \in \mathcal{R}^m$: error term of observation equation.
- $\boldsymbol{s}(t) = (s^{(n_1)}(t), ..., s^{(n_m)}(t)) \in \mathcal{R}^m$: credit spread at the time of t for n_i $(i=1, ..., m)$ time to maturity.
- $\boldsymbol{F} \in \mathcal{R}^{m\times 3}$: coefficient matrix of the term structure factor. Element by element notation is as follows.

$$\boldsymbol{F} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda n_1}}{\lambda n_1} & \frac{1-e^{-\lambda n_1}}{\lambda n_1} - e^{-\lambda n_1} \\ 1 & \frac{1-e^{-\lambda n_2}}{\lambda n_2} & \frac{1-e^{-\lambda n_2}}{\lambda n_2} - e^{-\lambda n_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda n_m}}{\lambda n_m} & \frac{1-e^{-\lambda n_m}}{\lambda n_m} - e^{-\lambda n_m} \end{pmatrix}. \tag{1}$$

- $\lambda \in \mathcal{R}$: parameters which determine the term structure factor.
- $\boldsymbol{H} \in \mathcal{R}^{3\times 3}$: covariance matrix of observation error ε. element by element notation is as follows.

$$\boldsymbol{H} = \begin{pmatrix} \sigma_\varepsilon^2(n_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_\varepsilon^2(n_m) \end{pmatrix}. \tag{2}$$

The above setting leads to the following state − space representation.

Transition Equation

$$\beta(t) - \mu = \boldsymbol{A}(\beta(t-1) - \mu) + \eta(t), \tag{3}$$
$$\eta(t) \sim N(0, \boldsymbol{Q}). \tag{4}$$

Measurement Equation

$$\boldsymbol{s}(t) = \boldsymbol{F}\beta(t) + \varepsilon(t), \tag{5}$$
$$\varepsilon(t) \sim N(0, \boldsymbol{H}). \tag{6}$$

DNS model in which three term structure factors are mutually independent is abbreviated to

DNS(indep). Then based on DNS(indep) the model mean reversion vector μ shifts between two regimes is constructed[4 5]. Mathematical notation is as follows.

- $S(t) \in \{0,1\}$: variables representing regime at time t.
- $\mu^k = (\mu^k_{\beta_1}, \mu^k_{\beta_2}, \mu^k_{\beta_3})' \in \mathcal{R}^3 (k=0,1)$ \$: mean reversion vector with regard to k.
- The transition probability is constant over time:

$$p_{ik} = Pr[S(t) = k \mid S(t-1) = i]. \tag{7}$$

The transition probability matrix is as follows.

$$\boldsymbol{P}^Z = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \in \mathcal{R}^{2\times 2}. \tag{8}$$

Based on the above setting the state-space representation is described as follows.

Transition Equation

$$\beta(t) - \mu^{S(t)} = \boldsymbol{A}(\beta(t-1) - \mu^{S(t)}) + \eta(t). \tag{9}$$

Measurement Equation

$$\boldsymbol{s}(t) = \boldsymbol{F}\beta(t) + \varepsilon(t). \tag{10}$$

Dynamic Nelson Siegel model with regime shifts is abbreviated to DNSRS model. According to Kim and Nelson [1999], the subsection describes an algorithm and estimation methodology of state-space mode including regime shifts. The parameters sets to be estimated θ are described as follows.

- $\theta^{DNSRS} = (\mu^k, \lambda, \boldsymbol{A}, \boldsymbol{Q}, p, q, \boldsymbol{H})$.

The following is the variables of the state-space model and dimension at each time step.

- ψ_t: denotes the vector of information set (credit spreads) available as of time t. ψ_t and $S(t)$ are mutually independent.
- $\beta_{t|t-1} = E[\beta_t \mid \psi_{t-1}] \in \mathcal{R}^3$: expectation(estimate) of term structure factor β as of t based on information set ψ_{t-1}.
- $\beta_{t|t} = E[\beta_t \mid \psi_t] \in \mathcal{R}^3$: expectation(estimate) of the term structure factor β conditional on information set ψ_t up to t.
- $\boldsymbol{P}_{t|t-1} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})' \mid \psi_{t-1}] \in \mathcal{R}^{3\times 3}$: covariance matrix of the term structure factor β conditional on information set ψ_t up to t.
- $\boldsymbol{P}_{t|t} = E[(\beta_t - \beta_{t|t})(\beta_t - \beta_{t|t})' \mid \psi_t] \in \mathcal{R}^{3\times 3}$: covariance matrix of the term structure factor β conditional on information set ψ_t up to t.
- $\boldsymbol{f}_{t|t-1} \in \mathcal{R}^{m\times m}$: conditional variance of the prediction error $\varepsilon^{(i,k)}_{t|t-1}$ conditional on information set ψ_t.

---

[4] AR matrix and the variance of the term structure factor do not depend on regimes.

[5] The previous research such as Bernadell, Coche and Nyholm [2005], Nyholm [2007] propose the model in which the mean reversion of the 'slope' factor only depends on regime and apply it to US treasury note. On the other hand the model in this study shifts does not only mean reversion of 'level' and but also 'slope' and 'curvature' in two regime in order to enhance the goodness fit.

- $\beta^{i}_{t|t-1} = E[(\beta_t | \psi_{t-1}, S(t-1) = i] \in \mathcal{R}^3$: expectation(estimate) of the term structure factor β conditional on information up to t based on information set ψ_t and $S(t-1) = i$.
- $\beta^{k}_{t|t-1} = E[(\beta_t | \psi_{t-1}, S(t) = k] \in \mathcal{R}^3$: expectation(estimate) of the term structure factor β conditional on information up to t based on information set ψ_{t-1} and $S(t) = k$.
- $\beta^{(i,k)}_{t|t-1} = E[(\beta_t | \psi_{t-1}, S(t) = k, S(t-1) = i] \in \mathcal{R}^3$: expectation(estimate) of the term structure factor β conditional on information up to t based on information set ψ_{t-1} and $S(t-1) = i$.
- $\varepsilon^{(i,k)}_{t|t-1} = s(t) - \boldsymbol{F}\beta^{(i,k)}_{t|t-1} \in \mathcal{R}^m$: prediction error β conditional on information up to t based on information set ψ_{t-1} and $S(t) = k$.

The following describes the algorithm of parameter estimation of state-space model.

1. Setting initial value.
   - $\beta^{k}_{1|1} = \mu^k$
   - $\mathrm{vec}(\boldsymbol{P}_{1|1}) = (\boldsymbol{I} - \boldsymbol{A} \otimes \boldsymbol{A})^{-1}\,\mathrm{vec}(\boldsymbol{Q})$.
   - Initial probability $Pr(S_1 = i|\psi_1)$: Calculate the steady-state or unconditional probability π_k $(k = 0, 1)$ and set the initial probability.

$$Pr[S(1) = 0|\psi_1] = \frac{1-q}{2-p-q},\ Pr[S(1) = 1|\psi_1] = \frac{1-q}{2-p-q}. \tag{11}$$

2. Do Step 3～12 for $t = 2, ..., T$.
3. Form a forecast of the unobserved term structure factor β conditional on information ψ_{t-1} up to t.

$$\beta^{(i,k)}_{t|t-1} = (\boldsymbol{I} - \boldsymbol{A})\mu^k + \boldsymbol{A}\beta^{i}_{t-1|t-1},\ i, k = 0,1 \tag{12}$$

4. Similarly calculate the mean squared error of the forecast.

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{A}\boldsymbol{P}_{t-1|t-1}\boldsymbol{A}' + \boldsymbol{Q}. \tag{13}$$

5. Calculate conditional variance of the prediction error $\varepsilon^{(i,k)}_{t|t-1}$ conditional on information set ψ_t.

$$\varepsilon^{(i,k)}_{t|t-1} = \boldsymbol{s}(t) - \boldsymbol{F}\beta^{(i,k)}_{t|t-1},\ i, k = 0, 1. \tag{14}$$

6. Calculate conditional variance of the prediction error $\varepsilon_{t|t-1}$.

$$f_{t|t-1} = \boldsymbol{F}\boldsymbol{P}_{t|t-1}\boldsymbol{F}' + \boldsymbol{H}. \tag{15}$$

   Regime-dependent term structure factor and regime non-dependent mean squared error of the forecast is updated using the following formula.

$$\beta^{(i,k)}_{t|t} = \beta^{(i,k)}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{F}'\boldsymbol{f}^{-1}_{t|t-1}\varepsilon^{(i,k)}_{t|t-1},\ i, k = 0, 1. \tag{16}$$

$$\boldsymbol{P}_{t|t} = (1 - \boldsymbol{P}_{t|t-1}\boldsymbol{F}'\boldsymbol{f}^{-1}_{t|t-1}\boldsymbol{F})\boldsymbol{P}_{t|t-1}. \tag{17}$$

7. Calculate the transition probability $Pr[S(t) = k, S(t-1) = i|\psi_{t-1}] \in \mathcal{R}$ at time $t-1$.

$$Pr[S(t) = k, S(t-1) = i|\psi_{t-1}] = p_{ik} \times Pr[S(t-1) = i|\psi_{t-1}],\ i, k = 0, 1. \tag{18}$$

8. Consider the joint density $f(\boldsymbol{s}(t), S(t)=k, S(t-1)=i|\psi_{t-1}) \in \mathcal{R}$.

$$f(\boldsymbol{s}(t), S(t)=k, S(t-1)=i|\psi_{t-1}) = f(\boldsymbol{s}(t)|S(t)=k, S(t-1)=i, \psi_{t-1}) Pr\ [S(t)=k, S(t-1)=i,|\psi_{t-1}]. \tag{19}$$

where the conditional density $f(\boldsymbol{s}(t)|S(t)=k,\ S(t-1)=i,\ \psi_{t-1})$ is obtained based on the prediction error decomposition.

$$f(\boldsymbol{s}(t)|S(t)=k, S(t-1)=i, \psi_{t-1}) = (2\pi)^{-\frac{m}{2}} |\boldsymbol{f}_{t|t-1}|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\varepsilon_{t|t-1}^{(i,k)\prime}(\boldsymbol{f}_{t|t-1})^{-1}\varepsilon_{t|t-1}^{(i,k)}\right). \tag{20}$$

9. Obtain marginal density $f(\boldsymbol{s}(t)|\psi_{t-1}) \in \mathcal{R}$ by the following formula.

$$f(\boldsymbol{s}(t)|\psi_{t-1}) = \sum_{i=0}^{1}\sum_{k=0}^{1} f(\boldsymbol{s}(t)|S(t)=k, S(t-1)=i, \psi_{t-1})\ Pr[S(t)=k, S(t-1)=i,|\psi_{t-1}]. \tag{21}$$

10. Once $s(t)$ is observed at the end of time t, the probability is updated.

$$Pr[S(t)=k, S(t-1)=i|\psi_t] = \frac{f(\boldsymbol{s}(t), \boldsymbol{s}(t)=k, S(t-1)=i|\psi_{t-1})}{f(\boldsymbol{s}(t)|\psi_{t-1})}, i, k=0, 1. \tag{22}$$

11. Calculate the filtered probability $Pr[S(t)=k|\psi_t] \in \mathcal{R}$.

$$Pr[S(t)=k|\psi_t] = \sum_{i=0}^{1} Pr[S(t)=k, S(t-1)=i|\psi_t],\ i=0, 1. \tag{23}$$

12. Calculate $\beta_{t|t}^{k}$.

$$\beta_{t|t}^{k} = \frac{\sum_{i=0}^{1} Pr[S(t)=k, S(t-1)=i|\psi_t]\ \ \beta_{t|t}^{(i,k)}}{Pr(S(t)=k|\psi_t)},\ \ i, k=0, 1. \tag{24}$$

Log likelihood of parameter set θ is calculated.

$$\sum_{t=1}^{T} \log(f(\boldsymbol{s}(t)|\psi_{t-1})). \tag{25}$$

Find parameter θ set to maximize the log likelihood. Calculate $\beta_{t|t}$ and the estimated credit spreads $\hat{\boldsymbol{s}}(t)$ using estimated $\beta_{t|t}^{k}$.

$$\beta_{t|t} = \sum_{k=0}^{1} Pr[S(t)=k|\psi_t]\beta_{t|t}^{k}, \tag{26}$$

$$\hat{\boldsymbol{s}}(t) = \boldsymbol{F}\beta_{t|t}. \tag{27}$$

Calculate the smoothed probability $Pr[S(t)=k|\psi_T] \in \mathcal{R}$ at time T using all the information in the sample under the given parameter set θ.

Do step 1～2 for $t=T-1, .., 1$.

1. Calculate filtered probability $Pr[S(t)=k, S_{t+1}=i|\psi_T]$.

$$Pr[S(t)=k, S(t-1)=i|\psi_T] = \frac{Pr[S_{t+1}=i|\psi_T]Pr[S(t)=k|\psi_t]p_{ki}}{Pr[S_{t+1}=i|\psi_t]},\ \ k, i=0, 1. \tag{28}$$

2. Calculate smoothed probability $Pr[S(t)=k|\psi_T]$.

$$Pr[S(t)=k|\psi_T] = \sum_{i=0}^{1} Pr[S(t)=k, S(t-1)=i|\psi_T],\ \ k=0, 1. \tag{29}$$

3 Estimation results

This chapter describes estimation results of DNSRS. First, interpretation of regime probability is discussed. Second, estimation results of parameter estimates are demonstrated and then goodness fit of the models is evaluated by AIC.

3.1 Regime identification: Interpretation of regime probability

This chapter examines the meaning of the estimated regime probability with regard to the shape of the term structure of credit spreads. The following three assumptions are built up regarding regimes to examine the relationship between the estimated regime probability and the term structure of credit spreads.

1. The regime with high value of mean reversion of level factor μ_{β_1} is regime 0 and the regime with low value of mean reversion of level factor μ_{β_1} is regime 1.
2. The regime with high value of mean reversion of slope factor μ_{β_2} is regime 0 and the regime with low value of mean reversion of slope factor μ_{β_2} is regime 1.
3. The regime with high value of mean reversion of curvature factor μ_{β_3} is regime 0 and the regime with low value of mean reversion of curvature factor μ_{β_3} is regime 1.

Under assumption 1 it is expected that in regime 0 the level of term structure of credit spreads is high while in regime 1 the level of term structure of credit spreads is low.

As for slope in regime 0 the slope of term structure of credit spreads is expected to be flat while in regime 1 the slope of term structure of credit spreads is steep.

As for curvature in regime 0 the curvature of term structure of credit spreads is assumed to be convex while in regime 1 the curvature of term structure of credit spreads is concave.

Under the above assumptions, individual firm is classified in a way that it belongs to regime 0 under the smoothed probability $Pr[S(t)=0|\psi_T]> 0.5$ and it belongs to regime 1 under the smoothed probability $Pr[S(t)=0|\psi_T]< 0.5$.

Figure 1, 2 and 3 show the average of the term structure of credit spread for level slope and

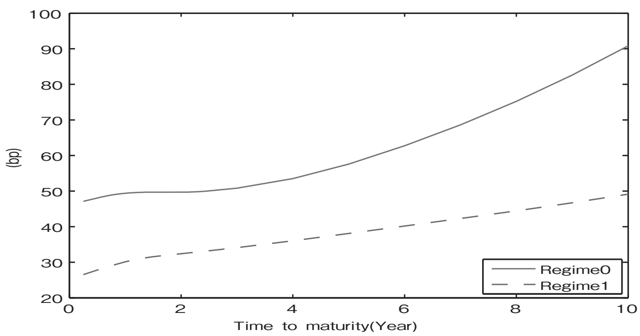

Figure 1 term structure of credit spread by regime (Identify regime by μ_{β_1})

Note: Assuming high value of μ_{β_1} is regime 0 and the low value is regime 1, individual firm is classified into regime 0 and regime 1 and take an average of the sample.

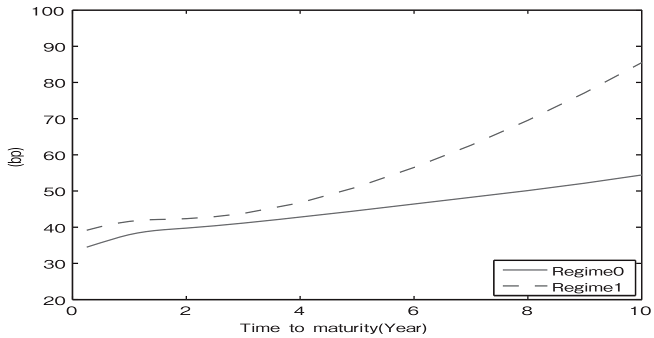

Figure 2 term structure of credit spread by regime（Identify regime by μ_{β_2}）

Note: Assuming high value of μ_{β_2} is regime 0 and the low value is regime 1, individual firm is classified into regime 0 and regime 1 and an average of the sample is taken.

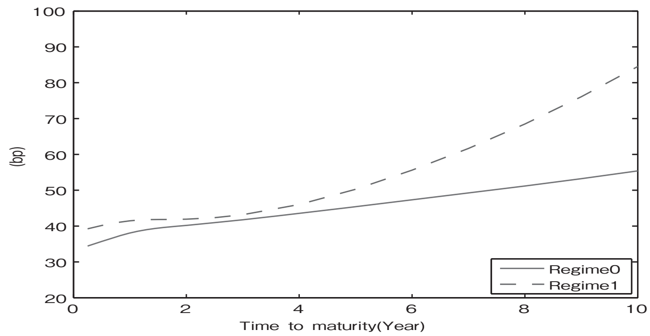

Figure 3 term structure of credit spread by regime（Identify regime by μ_{β_3}）

Note: Assuming high value of μ_{β_3} is regime 0 and the low value is regime 1, individual firm is classified into regime 0 and regime 1 and an average of the sample is taken.

curvature by using above methods.

If the regime is classified by the level μ_{β_1} (Chart l), it indicates high credit spreads for regime 0 and low credit spreads for regime 1 while under regime 0 the curve becomes steep especially for the long end. If the regime is classified by the slope μ_{β_2} (Chart 2), it indicates flat credit spreads for regime 0 and steep credit spreads for regime 1 If the regime is classified by the curvature μ_{β_3} (Chart 3), it indicates concave credit curve and upward steep credit curve for regime 1. Given the above results it seems natural to conclude that the regime of DNSRS model might suggest ‘slope’ shifts among the driving factors of credit spreads.

3.2 Examination of parameter estimates of DNSRS model

This section describes estimation results of 56 firms. Table I indicates estimation results of mean reversion parameters of DNSRS model. Table I shows that regime 0 and 1 is classified based on the previous chapter and shows the mean, the median, the first quartile and the third quartile. Table 1 shows each sign of $\mu^0_{\beta_1}$, $\mu^1_{\beta_1}$, $\mu^0_{\beta_2}$, $\mu^1_{\beta_2}$, $\mu^0_{\beta_3}$, $\mu^1_{\beta_3}$ indicated the opposite direction.

Table 1 Parameter estimates：DNSRS model

	mean	median	the fisrt quartile	the third quartile
$\mu_{\beta_1}^0$	0.054	0.047	0.029	0.084
$\mu_{\beta_1}^1$	−0.041	−0.037	−0.078	−0.003
$\mu_{\beta_2}^0$	0.045	0.039	0.010	0.096
$\mu_{\beta_2}^1$	−0.051	−0.045	−0.082	−0.023
$\mu_{\beta_3}^0$	0.059	0.063	0.017	0.100
$\mu_{\beta_3}^1$	−0.063	−0.065	−0.100	−0.038

Note 1: Estimation results of μ parameter of DNSRS model.

Note 2: The regime with high value of μ_{β_2} is considered to be regime 0, the regime with low value is considered to be regime 1 and the mean, the median, the first quartile and the third quartile are calculated

This result shows in the regime with the high value of μ_{β_2}, flat shape of the term structure of credit spreads, there are a lot of firms which level of credit spreads is high and the curvature is convex. On the other hand in the regime with the low value of μ_{β_2}, steep shape of the term structure of credit spreads, there are a lot of firms which level of credit spreads is low and the curvature is concave.

3.3 Regime probability and macro economic environment

This section investigates the time series of estimated regime probability. The left axis of Figure 4 shows the mean of 56 firms of times series of smoothed probability of regime 1 for DNSRS model The right axis of Figure 4 shows NIKKEI225 as a proxy for economic sentiment.

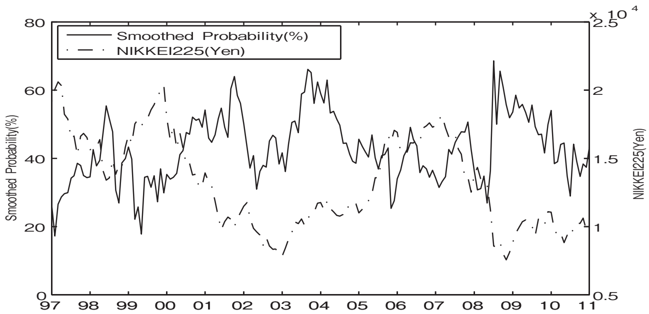

Figure 4 Smoothed probability and NIKKEI225

Note: The left axis shows the mean of 56 firms of times series of smoothed probabilities of regime 1 for NSRS(fp) model. The right axis shows NIKKEI225.

It can be seen from Figure 4 that the smoothed probability and NIKKEI225 is correlated. Actually the correlation coefficient is about −0.6. Especially it is obvious that the smoothed probability increased during the financial crises in the first half of 2000 and in the second half of 2000. When we look back at the corporate bond market in the first half of 2000, we encountered a series of event such as MYCAL default, bankruptcy of Enron Corp and synchronized terrorist attacks which leds to deterioration of market sentiments. Firms decreased their production and profits. In the financial crisis of the second half of 2000 the stock index dropped sharply after the Lehman Brothers bankruptcy. Reflecting the deterioration of business and market sentiment, the corporate bond investors tended to sell off the risky asset and their spread became widened sharply. Such investor's attitude toward risk aversion led to a sell off especially for the long end of the credit curve and the term structure of credit spreads is considered to be steepening.

3.4 Model evaluation

This section investigate the goodness fit of sample data for the DNS(indep) and DNSRS. We calculate the estimation error of credit spreads of individual firms and calculate the ration of a number of firms which have a smaller estimated error than that of DNS(indep). Finally I compare the model based on AIC. Table 2 shows RMSE of the term structure of credit spreads by time to maturity. Median of the whole sample is demonstrated as basis point. In addition the number of firms in which RMSE is lower than the DNS model is summed up and divided by the whole sample. The larger the ratio become, the more the firms with the superior predictive power against the DNS(indep).

Table 2 RMSE and AIC by model

	DNS(indep)	DNSRS	
Time to maturiy (year)	RMSE	RMSE	Ratio
1	1.57	1.65	39.3%
3	1.72	1.76	60.7%
5	2.33	2.18	69.9%
7	2.00	2.00	55.4%
8	1.59	1.63	60.7%
AIC	−	−	100.0%

Note 1: RMSE and AIC of the term structure model is calculated. The median of the whole sample is demonstrated as basis point. In addition, the number of firms in which RMSE is lower than the DNS model is summed up and divided by the whole sample. The larger the ratio become, the more the firms with the superior predictive power against the DNS(indep)

Note 2: In the bottom of table 2 the ratio of the number of the firm of DNSRS model with lower AIC to DNS(indep) model is calculated. The larger the ratio become the better the goodness fit of the model become. The ratio of DNSRS/DNS(indep) 100% shows the whole sample of DNS(indep) shows the smaller AIC.

Considering the median of RMSE by time to maturity, there is no such difference between DNS(indep) and DNSRS. In terms of ratio of the number of firms at the time of maturity, the number of firms which have a lower estimation error than DNS(indep), amounts to about 40 to 60%. As the time to maturity becomes longer, the number of the firms with lower estimation error increases. At the bottom of the table the ratio of the number of the firm of the DNSRS model with lower AIC to DNS(indep) model. The larger the ratio becomes the better the goodness fit of the model becomes. The ratio of DNSRS/DNS(indep) 100% shows the whole sample of DNS(indep) shows the smaller AIC. This result demonstrates the DNSRS model has better goodness fit than DNS(indep) in terms of information criterion.

The above results lead to the conclusion that the term structure model with regime shifts has strong in-sample performance.

4 Concluding remarks

This study develops a regime-switching extension of the dynamic Nelson-Siegel and demonstrates its estimation methodology. The models are estimated using Japanese corporate bond spreads term structure data on an individual firm basis. The conclusion is summarized as below. First, it can be said that the regime of the DNSRS model might suggest a 'slope' shift among the driving factors of credit spreads. Second, the estimation results indicate estimated regime probability is closely linked to business and market sentiment. Third, the term structure model with regime shifts has a superior in-sample fit than the model without regime shifts.

The new finding and practical usefulness of the study make a contribution to application to the fixed income financial product such as emerging sovereign bond yield or Credit Default Swap which has nonlinear properties of time series under a sovereign debt crisis. A further study of regime-switching term structure model should be conducted by developing the model which transition probability is time-varying and driven by macro economic variables. Further study of application for the model such as forecasting credit spread and investment strategy should be conducted. One of the limitations of this study is to show the goodness fit of the model by aggregating estimation results of individual firms and avoiding the difference of credit quality between firms. To overcome this extending the term structure model including a hierarchical structure in which term structure factors depend on both global factors and firm-specific factors should be studied further.

References

[1] Alexander, C. and A. Kaeck [2008], "Regime dependent determinats of credit spreads," *Journal of Banking and Finance* 32, 1008–1021.
[2] Bansal, R. and H. Zhou [2002]. "Term Structure of Interest Rates with Regime Shifts," *Journal of Finance* 57, 1997–2043.
[3] Bernadell, C., J. Coche, and K. Nyholm [2005], "Yield Curve Prediction for the Strategic Investor", *ECB Working paper series*, 472.
[4] Dai, Q. and K. J. Singleton [2003], "Specification Analysis of Affine Term Structure Models," *Journal of Finance* 55, 1943–1978.
[5] Dai, Q., K. J. Singleton and Yang [2007], "Regime Shifts in a Dynamic Term Strcuture Model of US Treasury Bond Yields," *The Review of Financial Studies* 20(5), 1669–1706.
[6] Diebold, F. X. and C. Li [2006], "Forecasting the Term Structure of Government Bond Yields," *Journal of Econometrics* 130, 337–364.
[7] Dionne, G., G., Gauthier, K. Hammami, M. Maurice,, and, J. G., Simonato [2011], "A Reduced form model of default spreads with Markov-swiching macroeconomic factors," *Journal of Banking and Finance* 35, 1984–2000
[8] Fisher, M., D. Nychka, and D. Zervos [1995], "Fitting the Term Structure of Interest Rates with Smoothing

Splines," *Finance and Economics Discussion Series, Federal Reserve Board*, 95−1.

[9] Garcia, R. and P. Perron [1996], "An Analysis of the Real Interest Rate Under Regime Shifts," *Review of Economics and Statistics*, 111−125.

[10] Gray, S. [1996], "Modeling the Conditional Distribution of Interest Rates as a Regime Switching Process," *Journal of Financial Economics* 42, 27−62.

[11] Hamilton, J. [1989], "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica* 57, 357−384. S, M. Priebsch and K. J. Singleton [2009], "Risk Premium Accounting in Macro-Dynamic Term Structure Models," *Working Paper*.

[12] Kim, C. J. and C. R. Nelson [1999], *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT Press.

[13] Landen, C. [2000]. "Bond Pricing in a Hidden Markov Model of the Short Rate," *Finance and Stochastics* 4, 371−389.

[14] Litterman, R. and J. Scheinkman [1991], "Common Factors Affecting Bond Returns", *Jounal of Fixed Income* 1(1), 54−61.

[15] Maalaoud, Dionne and Francois [2009], "Credit Spread Changes within Switching Regimes," *Working Paper*.

[16] Nelson, C., and A. Siegel [1987], "Parsimonious Modeling of Yield Curves," *Journal of Business* 60, 473−489.

[17] Nyholm. K and R, Vidova-Koleva [2008], "Nelson-Siegel, Affine and Quadratic yield curve specifications: which one is better at forecasting?," *Working Paper*.

[18] Nyholm, K. [2007], "A New Approach to Predicting Recessions," *EconomicNotes*, 36(1), 27−42.

[19] Steeley, J. M. [1991], "Estimating the Gilt-edged Term Structure: Basis Splines and Confidence Intervals," *Journal of Business Finance and Accounting*, 18(4), 513−529.

[20] Zhu, X., and S. Rahman [2009], "A regime switching macro finance model of the term structure," *Working Paper*.